Performance Analysis of Distributed Space-Time Block-Encoded Sensor Networks

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Abstract—Sensor networks are comprised of nodes with minimal processing and radio-frequency functionalities. In such networks, it is assumed that a source sensor communicates with a target sensor over a number of relaying sensors by utilizing distributed but cooperative low-complexity space-time encoding techniques, thereby achieving highly robust communication links. Each relaying stage is hence comprised of a given number of cooperating sensor nodes, which may or may not exchange additional data. The contribution of this paper is the derivation of throughput-maximizing resource-allocation strategies for various sensor network configurations. The case of full data exchange at each relaying stage is analyzed first and then relaxed to the case of partial data exchange. Monte Carlo simulations are used to numerically verify the theoretically derived performance of distributed sensor networks. It is shown that notable power savings can be achieved compared to traditional single-link and nonoptimized sensor networks.

Index Terms—Distributed information systems, multiple-input–multiple-output (MIMO) systems, sensor networks.

I. INTRODUCTION

The function of sensors is to sense certain features of their surroundings and pass this information to a sensor or a unit that is capable of processing such data. Sensor networks are significantly different from traditional ad hoc networks. First, the number of sensor nodes in a sensor network can be several orders of magnitude higher than the number of nodes in an ad hoc network. Moreover, sensor nodes are usually densely deployed, prone to failures, and limited in power provision, computational complexity, and memory [1].

Majority of sensors are stationary; however, some communication paths between them may potentially be permanently or frequently obstructed. This may effect the throughput and stability of the routing path from the source to the target sensors [2]. It is therefore a task of a sensor network designer to allow for robust data transmission with minimal power consumption and complexity. Hence, the aim of this paper is to analyze and optimize the performance of sensor networks for the topologies that will be described later.

Background: To date, a considerable amount of work has been dedicated to the conceptual design, information and communication theoretic analysis, and practical implementation of sensor networks and underlying communication paradigms.

For practical demonstrations, the landmark Wireless Integrated Network Sensors (WINS) [3] and the SmartDust [4] projects, which aimed to integrate sensing, computing, and wireless communication capabilities into a small form factor to enable low-cost production of tiny sensor nodes in large numbers, are worth mentioning [5]. In the meantime, innumerable other projects that aimed to investigate various practical issues related to sensor network deployment have appeared; for example, France Télécom’s participation in the IST EU project Pervasive Ultrawideband Low-Spectral-Energy Radio Systems (PULSERS) have resulted in functioning sensor networks [6].

For the conceptual proposition of communication topologies tailored to the requirements of sensor networks, the landmark contribution [1], which highlights prior art and research challenges, is the most notable. One of the major design constraints mentioned in [1] has been the requirement on low complexity and fairly high robustness, which triggered distributed and cooperative data relaying topologies to be applied to sensor networks. The process of data relaying, i.e., the transmission of data from source to destination via at least one intermediate node, is known to yield information theoretical gains [7]–[14] and also drastic power savings due to the nonlinear behavior of the aggregate pathloss [15]. Simple cooperation between the relaying nodes is known to provide additional diversity and, hence, boost the capacity and performance further, as demonstrated in [16]–[20]. Furthermore, the process of distributed and cooperative data processing abates the limitation of having only one antenna element per sensor [21]–[32], thereby factually creating a multiple-input–multiple-output (MIMO) channel and yielding the associated capacity and performance gains [33]–[39].

Although the method of relaying has been introduced in 1971 by van der Meulen [7] and has also been studied in [8], the first rigorous information theoretical analysis of the relay channel has been exposed in [9] and [10]. The capacity of such a simple three-terminal relaying configuration was shown to exceed the capacity of a direct link, where only Gaussian communication channels have been assumed. This paper has been extended in [11] and [23], where the information theoretically offered throughput per terminal in a large-scale relaying network has been analyzed. In [21], the information theoretical results for
distributed space-time channels with possible feedback have been utilized to design simple communication protocols, taking into account systems with and without temporal diversity as well as various forms of cooperation. It has been demonstrated that cooperation yields full spatial diversity, which allows drastic transmit power savings at the same level of outage probability for a given communication rate.

For communication theory, the work of Sendonaris et al. [16] has been a milestone contribution where a very simple but effective user cooperation protocol has been suggested to boost the uplink capacity and lower the uplink outage probability for a given rate. The simple cooperative protocol has been extended by Sendonaris et al. to more sophisticated schemes, which can be found in the excellent contributions [17] and [18]. The contribution [25] is a conceptual and mathematical extension to [16], where energy-efficient multiple-access protocols are suggested based on decode-and-forward and amplify-and-forward relaying technologies.

The error rate, throughput, and other performance metrics of distributed cooperative relaying networks have been analyzed in [32] and [40]–[57]. The analysis in [52] facilitates asymptotic error rates of generic cooperative topologies to be characterized in closed form. This analysis has then been extended in [54] to obtain the error rates of distributed space-time block-encoded cooperative relaying systems as well as the algorithms that allocate optimum power to the relaying nodes under conditions that are different from those in this paper.

**Contributions:** The contributions of this paper can be summarized as follows.

1. The symbol error rates (SERs) over generalized Nakagami-fading channels have been derived in closed form, where each subchannel may obey different statistics.
2. The asymptotic end-to-end bit error rates (BERs) are derived in closed form for a generic distributed and cooperative relaying topology.
3. This facilitates low-complexity throughput-maximizing resource-allocation strategies to be derived in closed form, where adaptation is performed on power, frame duration, and modulation index.
4. Finally, the performance of the aforementioned developed strategies is assessed for a wide variety of sensor network topologies.

**Assumptions:** It is assumed that each sensor is in possession of one antenna element only, and the transceiver utilizes a time-division multiple-access (TDMA) protocol. In TDMA, each sensor receives data over the entire frequency band $W$ and a frame duration $T_1$. After possible processing, the data is retransmitted over the entire frequency band $W$ and in a frame duration $T_2$, during which time, it is not capable of receiving any data.

It is also assumed that an appropriate medium-access control is deployed, which guarantees that at any time, only one transmission link from the source to the target sensors is active. It is also assumed that the protocol facilitates the exchange of some nonfrequent control data between the sensors, e.g., the average channel gains in the network, synchronization data, etc.

It is further assumed that spatially adjacent sensors are grouped such that they allow for a distributed MIMO reception and retransmission. Since the sensors are spatially separated, no correlation will be observed. Sensors belonging to the same relaying stage may or may not exchange additional data among each other. Note that the choice of grouping is beyond this paper; however, the exposed analysis can certainly aid the design of optimum grouping and cooperation strategies.

Note that the TDMA protocol does not allow immediate relaying of the data stream, i.e., the data has to be down-converted to baseband, amplified, stored, and upconverted before retransmission. To save memory, a hard decision is performed on the data stream prior to storage, whose complexity and power consumption is negligible compared to the other processes. Furthermore, the load/store operations are known to consume a nonnegligible amount of power, which is mainly due to the fairly high capacitance of the data busses [58], [59]. To minimize the amount of load/store operations in a sensor, baseband data processing ought to be avoided where possible; this includes operations such as channel coding and cyclic redundancy check (CRC).

To reflect the cheap and low-complexity hardware architecture of sensor nodes, the above-mentioned reasons stipulate the deployment of regenerative relaying and the decision on the correctness of a received packet to be drawn at the target processing unit only.

The aim of this paper is to optimize the throughput for a given total transmission power depending on the prevailing average channel conditions. All the average channel gains of the topology, be they slowly changing or constant, are assumed to be available within each sensor node, as they can be measured once or at low-duty cycles.

**Paper Structure:** In Section II, the system model is described in detail. The principle of a distributed (sensor) network as well as the encoding/decoding strategy in each sensor is explained. In Section III, the error rates of the distributed space-time block codes (STBCs) are exposed. These are then utilized in Sections IV and V to derive throughput-maximizing resource-allocation strategies, assuming full and partial cooperation at each relaying stage, respectively. Finally, conclusions are drawn in Section VI.

## II. System Model

### A. General Deployment

The subsequent description relates to a generalized deployment of distributed and cooperative space-time block-encoded multistage sensor networks, as depicted in Fig. 1. Here, a source sensor $s-S$ communicates with a target unit or target sensor $t-S$ via a given number of relaying sensors $r-Ss$. Spatially adjacent $r-Ss$ are grouped into virtual antenna arrays (VAA), thereby forming a cooperative relaying VAA ($r-VAA$) tier. The $s-S$ and $t-S$ themselves might be a member of a VAA, which are henceforth referred to as source VAA (s-VAA) and target VAA (t-VAA), respectively. The coordinated process of data relaying from the same tier is referred to as “cooperation” or “cooperative communication [21],” whereas the process of data relaying within the same tier is referred to as “data exchange.”
B. Distributed Space-Time Block Encoding and Decoding

Source Sensor: The s-S Gray maps $b_0$ source information bits onto symbol $x$ by utilizing an $M_0$-phase-shift keying (PSK) or $M_0$-quadratic-amplitude modulation (QAM) signal constellation, where $b_0 = \log_2 M_0$. The data stream is broadcast to the adjacent sensors.

First-Tier Relaying Sensors (s-VAA): The broadcasted data is received by the r-Ss belonging to the first-tier VAA. They detect, space-time encode, and transmit the data simultaneously with a total power $S_1$ over an allocated fractional frame duration $T_1$. Each r-S Gray maps $s_1 b_1$ bits onto symbols $x_1, x_2, \ldots, x_{s_1}$ by utilizing an $M_1$-PSK or $M_1$-QAM signal constellation, where $b_1 = \log_2 M_1$, and $s_1$ is the number of symbols per space-time encoding. The $\{x_k\}_{k=1}^{s_1}$ are encoded with an orthogonal space-time coding matrix $G_1$ of size $d_1 \times t_1$, where $d_1$ is the number of symbol durations required to transmit the space-time codeword, and $t_1$ is the number of distributed r-Ss (and therefore, equivalent to the actual number of transmit antennas). At each time instant $k$, the encoded symbols $c_{k,i}$ with $k = 1, \ldots, d_1$ and $i = 1, \ldots, t_1$ are transmitted simultaneously from the $i$th distributed r-S. Clearly, the rate of the first-tier STBC is $R_1 = s_1/d_1$.

Second-Tier Relaying Sensors: The second relaying tier is formed by $q_2$ spatially adjacent sensors, with one antenna each. Some sensors may exchange received data among each other, thereby forming $Q_2$ clusters, where $1 \leq Q_2 \leq q_2$. The case of $Q_2 = 1$ represents the scenario where all sensors cooperate, whereas $Q_2 = q_2$ means that none of the sensors cooperate. The $j$th cluster is assumed to consist of $r_{2,j}$ sensors, thereby achieving $r_{2,j}$ receive antennas, where $1 \leq j \leq Q_2$ and $q_2 = \sum_{j=1}^{Q_2} r_{2,j}$. Therefore, $Q_2$ space-time block-encoded MIMO channels are created, each with $t_2$ transmit antennas and $r_{2,j} \in [1, Q_2]$ receive antennas.

After data exchange, the data are space-time block decoded and reencoded according to a given orthogonal space-time coding matrix $G_2$ of size $d_2 \times t_2$ with a rate of $R_2$, where the encoding and transmission process is the same as that
previously described. The number of transmit elements \( t_2 \) is, at most, the number of sensors \( q_2 \) in the second tier.

**Tth-Tier Relaying Sensors (t-VAA):** The decoding/encoding process is continued until the VAA with the t-S is reached. The sensors in the Tth tier receive data from the previous tier. They relay the data to the target sensor.

**Target Sensor:** The t-S receives and space-time block decodes the data, after which it performs the final detection. If the s-S deploys a channel code, e.g., a simple trellis code, then the t-S performs the equivalent channel decoding (which has not been considered in this paper).

Each relaying sensor tier clearly may use a different signal constellation and STBC, i.e., it may adapt its modulation depending on the prevailing channel conditions. It is only of importance that the consecutive tier has knowledge of the transmission parameters of the previous tier.

### III. Error Rates of Distributed STBCs

The derivation of the fractional resource-allocation strategies relies on the error rates of the STBCs, where the subchannel gains from any transmit to any receive sensor node may vary. This is in contrast to traditional MIMO systems, where the subchannel gains are assumed to be equal. Therefore, the BERs, SERs, and frame error rates (FERs) for distributed STBCs over Rayleigh and Nakagami flat-fading channels with different channel statistics are given here. Note that the BERs and FERs are easily obtained from the SERs at high signal-to-noise ratios (SNRs).

#### A. SERs

The space-time encoding and decoding scheme, as depicted in Fig. 2, represents an arbitrary stage within the multistage sensor network introduced before. Here, an STBC of rate \( R \) is transmitted from a transmitter with \( t \) transmit elements and received by \( r \) receive elements. The flat-fading subchannel from the \( i \)th transmit element to the \( j \)th receive element is denoted as \( h_{(i-1)r+j} \), where \( 1 \leq i \leq t \) and \( 1 \leq j \leq r \). The \( u \triangleq t \times r \) subchannels have an average power \( \gamma_{(i-1)r+j} \triangleq \mathbb{E}[|h_{(i-1)r+j}|^2] \), where \( \mathbb{E}[\cdot] \) denotes the expectation.

**Rayleigh Fading—Equal Subchannel Gains:** According to [60], the SER for \( M \)-PSK over a MIMO channel with equal subchannel gains \( \gamma_1 = \cdots = \gamma_u \triangleq \gamma \) can be expressed in closed form as that given in (1), shown at the bottom of the page, where \( g_{\text{PSK}} \triangleq \sin^2(\pi/M) \), \( M \) is the modulation order, \( \Gamma(x) \) is the complete Gamma function, \( _2F_1(a, b; c; x) \) is the Gauss hypergeometric function with two parameters of type 1 and one parameter of type 2 [61, Sec. 9.14.1], and the function \( F_1(a, b; c; x, y) \) is the Appell hypergeometric function of two variables [61, Sec. 9.180.1]. To simplify the notation in subsequent analysis, the notation \( F_{\text{QAM}}(e) = P_{\text{QAM}}(u, t, R, \gamma, S/N, M) \) is adopted.

**Rayleigh Fading—Unequal Subchannel Gains:** For unequal subchannel gains, the SERs of the \( M \)-PSK and \( M \)-QAM

\[
P_{\text{PSK}}(e) = \frac{1}{\left(1 + \frac{g_{\text{PSK}} \gamma S}{R t N}\right)^u} \left[ \frac{1}{\sqrt{\pi}} \frac{\Gamma(u+1/2)}{\Gamma(u+1)} _2F_1\left( u, 1/2; u+1; \left(1 + \frac{g_{\text{PSK}} \gamma S}{R t N}\right)^{-1}\right) 
\right. \\
+ \frac{1}{\sqrt{\pi}} \frac{\Gamma(u+1/2)}{\Gamma(u+1)} _2F_1\left( u, 1/2; u+1; \frac{1}{1 + \frac{g_{\text{PSK}} \gamma S}{R t N}} \right) \\
\left. \left. + \sqrt{1 - g_{\text{PSK}} \gamma S} \right) \right]
\]

\[
P_{\text{QAM}}(e) = \frac{1}{\left(1 + \frac{g_{\text{QAM}} \gamma S}{R t N}\right)^u} \left[ \frac{2g_{\text{QAM}} \gamma S}{\pi} \frac{\Gamma(u+1/2)}{\Gamma(u+1)} _2F_1\left( u, 1/2; u+1; \left(1 + \frac{g_{\text{QAM}} \gamma S}{R t N}\right)^{-1}\right) 
\right. \\
- \frac{1}{\left(1 + \frac{g_{\text{QAM}} \gamma S}{R t N}\right)^u} \frac{2g_{\text{QAM}} \gamma S}{\pi(2u+1)} _2F_1\left( u, 1; u+3/2; \frac{1}{1 + \frac{g_{\text{QAM}} \gamma S}{R t N}} \right) \\
\left. \left. + \sqrt{1 - g_{\text{QAM}} \gamma S} \right) \right]
\]
modulation schemes have been derived in [32] as

\[ P_{PSK/QAM}(\epsilon) = \sum_{i=1}^{u} K_i \cdot P_{PSK/QAM}(1, t, R, \gamma_i, S/N, M) \]  

(3)

where the constants \( K_i \) are given as \( K_i = \prod_{j=1, j \neq i}^{u} \gamma_i / (\gamma_i - \gamma_j) \). As an example, Fig. 3 depicts the SER versus the SNR, in decibels, for the distributed Alamouti scheme with one receive antenna only. The power of the unequal channel coefficients is arbitrarily chosen such that \( \gamma_1 = 4/3 \) and \( \gamma_2 = 2/3 \). The cases where only the channel with power \( \gamma_1 \) is utilized, only the channel with power \( \gamma_2 \) is utilized, and the distributed Alamouti STBC is utilized, are depicted. The latter one is corroborated by numerical simulations.

Clearly, the distributed scenario provides diversity gain even for unbalanced channel gains, whereas the single links exhibit a less steep error curve. The gain of the distributed case at a SER of \( 10^{-5} \) is approximately 20 dB.

Rayleigh Fading—Generic Subchannel Gains: Generally, the channel gains \( \gamma_{i(1,u)} \) can be different where some gains are repeated. There shall be \( g \leq u \) distinct channel gains, which are hereinafter referred to as \( \gamma_i \), with each of them being repeated \( \nu_i \) times. In this case, the respective error rates can be expressed as [32]

\[ P_{PSK/QAM}(\epsilon) = \sum_{i=1}^{g} \sum_{j=1}^{\nu_i} K_{i,j} \cdot P_{PSK/QAM}(j, t, R, \gamma_i, S/N, M) \]  

(4)

where coefficients \( K_{i,j} \) are expressed as

\[ K_{i,j} = \frac{1}{(\nu_i - j)!} \left( -\frac{1}{R} \frac{\gamma_i S}{N} \right)^{\nu_i - j} \times \frac{\partial^{\nu_i - j}}{\partial s^{\nu_i - j}} \]  

\[ \times \left[ \prod_{j' = 1, j' \neq i}^{g} \left( 1 - \frac{1}{R} \frac{\gamma_i S}{N} \cdot s \right)^{\nu_i} \right]_{s = \left( \frac{1}{N} \frac{\gamma_i S}{N} \right)^{-1}} \]  

(5)

Nakagami Fading—Equal Subchannel Gains: The cases of Nakagami fading are similarly derived as the Rayleigh fading cases. Assuming a Nakagami-fading channel with equal subchannel gains \( \gamma_1 = \ldots = \gamma_u \) and equal fading parameters \( f_1 = \ldots f_u = f \), the respective SERs can be obtained as [32]

\[ P_{PSK/QAM}(\epsilon) = P_{PSK/QAM}(fu, ft, R, \gamma, S/N, M) \]  

(6)

Nakagami Fading—Unequal Subchannel Gains: Finally, the respective error rates for a Nakagami-fading channel with different subchannel gains \( \gamma_{i(1,u)} \) and different fading factors \( f_i \) can be expressed as [32]

\[ P_{PSK/QAM}(\epsilon) = \sum_{i=1}^{u} \sum_{j=1}^{\nu_i} K_{i,j} \cdot P_{PSK/QAM}(j, t, R, \gamma_i, S/N, M) \]  

(7)

where

\[ K_{i,j} = \frac{1}{(\nu_i - j)!} \left( -\frac{1}{R} \frac{\gamma_i S}{N} \right)^{\nu_i - j} \times \frac{\partial^{\nu_i - j}}{\partial s^{\nu_i - j}} \]  

\[ \times \left[ \prod_{j' = 1, j' \neq i}^{g} \left( 1 - \frac{1}{R} \frac{\gamma_i S}{N} \cdot s \right)^{\nu_i} \right]_{s = \left( \frac{1}{N} \frac{\gamma_i S}{N} \right)^{-1}} \]  

(8)

B. BERs

The derived dependences relate the average transmitted signal power to the SER. The exact BER of generic \( M-PSK \) and \( M-QAM \) schemes is, however, difficult to obtain. Analysis is greatly simplified if the bits are Gray mapped onto the symbol, i.e., adjacent symbol constellation points differ only by one bit [62]. In that case, the BER \( P_b(\epsilon) \) is easily related to the SER via [62]

\[ P_b(\epsilon) = \frac{P_s(\epsilon)}{\log_2(M)} \]  

(9)

at low error rates or sufficiently high SNRs.

C. Packet Error Rates (PERs)

A data packet is assumed to consist of \( B \) bits. A packet error occurs if at least one of the \( B \) bits is erroneous. With the assumptions of Section I, the bits in a packet are independent from each other, which yields for the PER

\[ P_b(\epsilon) = 1 - (1 - P_b(\epsilon))^B \]  

(10a)

\[ = 1 - BP_b(\epsilon) \]  

(10b)

where (10b) holds at low FERs or sufficiently high SNRs. Note that in the case of slow-fading channels, (10) can be approached with the aid of a suitable time interleaver.
IV. MAXIMUM THROUGHPUT FOR FULL DATA EXCHANGE

The error rates obtained in the previous section are utilized here to derive fractional resource-allocation rules, assuming that a decision on the correctness of the received data packet is done at the target processing unit. This should not be confused with transparent relaying, where the information is amplified and forwarded. It is also in contrast to a stage-by-stage detection, where a decision on the correctness of the received packet is done at each sensor-relaying stage.

If all the relaying sensors per stage exchange the received data and this takes places at a sufficiently high SNR, then the signal samples from the previous stage are the same for all the relaying sensors. Therefore, if an error occurs in the signal from the previous stage, that error is the same for all the relaying sensors belonging to the same stage. This applies not only to STBCs but also to any type of applied space-time coding.

Such a scenario provides a great simplification to the analysis since the errors in consecutive stages become independent. This is in contrast to a generic relaying process with no or partial data exchange (clustering), where one relaying cluster may have more reliable estimates than another relaying cluster in the same relaying tier VAA, leading to error dependence between the stages.

Subsequently, the problem of maximizing the end-to-end throughput is shown to be equivalent to the problem of minimizing the end-to-end BER. For the sake of clarity, the fractional resource-allocation rules are derived first for the cases of maximizing the end-to-end BER. For the sake of clarity, the fractional transmission power to be assigned to each relaying stage is determined depending on the previously derived fractional resource-allocation strategies that maximize the end-to-end throughput.

To this end, note that $P_{\text{e,2e}}(\epsilon)$ is a function of the error rate occurring at each relaying stage and hence depends on $M_{v \in (1,K)}$, $R_{v \in (1,K)}$, and the fractional transmission power allocated to each stage. Optimizing (11) with respect to these parameters is very complex, which is why the optimization process is performed in three stages.

First, the modulation indexes $M_{v \in (1,K)}$ are fixed (which is relaxed in the third optimization stage), and the asymptotic case where SNR $\rightarrow \infty$ is considered. This reduces (11) to

$$\Theta = \min_{v \in (1,K)} \{\alpha_v R_v \log_2(M_v)\} \tag{12}$$

where the fractional frame durations $\alpha_v$ need to be chosen to maximize $\Theta$ under constraint $\sum_{v=1}^{K} \alpha_v = 1$. This is achieved by equating all $\alpha_v R_v \log_2(M_v)$, which results in

$$\alpha_v' = \frac{\prod_{w=1, w \neq v}^{K} R_w \cdot \log_2(M_w)}{\sum_{k=1}^{K} \prod_{w=1, w \neq k}^{K} R_w \cdot \log_2(M_w)} \tag{13}$$

Second, the throughput in (11) is maximized if the end-to-end PER is minimized. With reference to (10b), this is clearly achieved by minimizing the end-to-end BER $P_{\text{e,2e}}(\epsilon)$, which requires optimum fractional transmission power to be assigned to each relaying stage. The BER at each stage is related with the occurring SER via (9), where for low error rates, one symbol error causes one bit error.

Third, the optimum modulation order $M_{v \in (1,K)}$ has to be determined depending on the previously derived fractional resource allocations. This is easily done by permuting all possible modulation orders at each stage to maximize the end-to-end throughput. Since the number of modulation orders will be limited, such optimization is feasible at low computational complexity.

Subsequently, the second step is performed assuming either total or partial (clustered) data exchange at each stage. The near-optimum fractional power allocation rules are first derived and then assessed in terms of their precision; finally, the maximum achievable throughput will be illustrated by means of a few examples.

A. Problem Simplification

It is assumed here that the s-S transmits $B$ bits per frame to the t-S via $K = T - 1$ relaying stages. The normalized end-to-end throughput (in bits per Hertz) from s-S to t-S can be expressed as

$$\Theta = \min_{v \in (1,K)} \{\alpha_v' R_v \log_2(M_v)\} \cdot (1 - P_{\text{e,2e}}(\epsilon)) \tag{11}$$

where $\alpha_v'$, $R_v$, and $M_v$ are the fractional frame duration, STBC rate, and modulation index of the $v$th stage, respectively, $P_{\text{e,2e}}(\epsilon)$ is the end-to-end FER, and $1 \leq v \leq K$.

Equation (11) has to be understood as follows: If there were no losses between a directly communicating s-S and t-S, then all of the $B$ bits reach the receiver; the throughput normalized by the total number of sent bits, hence, mounts to 1. The use of a modulation scheme with index $M$ and an STBC with rate $R$ during a fractional frame duration $\alpha'$ to accomplish such a link results in a throughput, which is normalized by the utilized time and bandwidth as $1 \cdot \alpha' \cdot R \cdot \log_2(M)$ [bits per second per Hertz]. For a communication system with $K$ relaying stages, the weakest link in the chain determines the throughput; hence, $\min_{v \in (1,K)} \{\alpha_v' R_v \log_2(M_v)\}$. Since the decision on the correctness of a data packet is done at the target processing unit and not in the relaying nodes, the term is further diminished by the loss caused by the end-to-end PER $P_{\text{e,2e}}(\epsilon)$. It is thus the aim to derive optimum resource-allocation strategies that maximize the end-to-end throughput.

B. Resource-Allocation Strategy

Under the assumption of full data exchange, each of the $K$ relaying stages experiences independent BERs, which are denoted here as $P_{b,v \in (1,K)}(\epsilon)$, caused by independent SERs $P_{s,v \in (1,K)}(\epsilon)$. A bit from the s-S is received correctly at the t-S only when at all stages, the bit has been transmitted correctly (the probability that two or more wrong bits may result again in a correct bit is approaching zero for sufficiently high SNRs).

The end-to-end BER can therefore be expressed as

$$P_{b,2e}(\epsilon) = 1 - \prod_{v=1}^{K} (1 - P_{b,v}(\epsilon)) \tag{14}$$
which, at low BERs or sufficiently high SNRs at every stage, can be written as

\[ P_{b,e2e}(e) = \sum_{v=1}^{K} P_{b,v}(e) \]

(15)

\[ = \sum_{v=1}^{K} \frac{P_{s,v}(e)}{\log_2(M_v)} \]

(16)

where \( M_v \) is the modulation order at the \( v \)th stage. Further analysis concentrates on the case of Rayleigh fading with equal channel gains per relaying stage; other cases are a straightforward extension to the exposed analysis, utilizing the expressions developed in Section III. Assuming that each stage is allocated a fractional power \( \beta_v \), the preceding dependence can be expressed as

\[ P_{b,e2e}(e) = \sum_{v=1}^{K} P_{s,v}(u_v, t_v, R_v, v, \gamma_t, \beta_v, S/N, M_v) \]

(17)

where the SERs \( P_{s,v}(\cdot) = P_{PSK/QAM}(\cdot) \) are given through (1) and (2), respectively. Furthermore, \( u_v = t_v \cdot r_v \), where \( t_v \) and \( r_v \) are the number of transmit and receive antennas in the \( v \)th stage, respectively, \( R_v \) is the rate of the STBC, \( \gamma_t \) is the average attenuation experienced, \( S \) is the total power allowed to deliver the information from source to sink, and \( N \) is the noise power.

The optimization process has only been performed with respect to the fractional power allocation \( \beta_v \). Even so, the optimization process is very intricate. To simplify the analysis further, an upper bound to the derived SERs for M-PSK and M-QAM is invoked. Following the analysis outlined in [63, Ch. 9], the SER for M-PSK in the \( v \)th relaying stage can be upper bounded as

\[ P_{s,v}(e) \leq \frac{M_v-1}{M_v} \left( 1 + \beta_v g_{PSK,v} \frac{\gamma_t S}{t_v N} \right) ^{-u_v} \]

(18)

where \( g_{PSK,v} = \sin^2(\pi/M_v) \). The end-to-end BER for an M-PSK scheme can hence be upper bounded as

\[ P_{b,e2e}(e) \leq \sum_{v=1}^{K} \frac{M_v-1}{M_v} \left( 1 + \beta_v g_{PSK,v} \frac{\gamma_t S}{t_v N} \right) ^{-u_v}. \]

(19)

Following a similar approach, the upper bound for the end-to-end BER of an M-QAM scheme can be derived as

\[ P_{b,e2e}(e) \leq \sum_{v=1}^{K} \frac{2q_v}{\log_2(M_v)} \left[ \left( 1 + \beta_v g_{QAM,v} \frac{\gamma_v S}{R_v t_v N} \right) ^{-u_v} + \frac{q_v}{2} \left( 1 + 2\beta_v g_{QAM,v} \frac{\gamma_v S}{R_v t_v N} \right) ^{-u_v} \right] \]

(20)

\[ \leq \sum_{v=1}^{K} \frac{2q_v}{\log_2(M_v)} \left( 1 + \beta_v g_{QAM,v} \frac{\gamma_v S}{R_v t_v N} \right) ^{-u_v} \]

(21)

where \( g_{QAM,v} = 3/2/(M_v - 1) \), and \( q_v = 1 - 1/\sqrt{M_v} \). Note that in (21), the second summand appearing in (20) was neglected due to \( q_v/2 < 1 \) and \( (1 + 2x)^{-u_v} \) being much less than \( (1 + x)^{-u_v} \) for a sufficiently large \( x \) and \( u_v \geq 1 \). Either modulation scheme results in an upper bound unified as

\[ P_{b,e2e}(e) \leq \sum_{v=1}^{K} A_v (1 + B_v \beta_v ^{-u_v}). \]

(22)

The constants \( A_v \) and \( B_v \) are obtained by comparing (22) with (19) or (21) to arrive at

\[ A_v = \begin{cases} \frac{M_v-1}{M_v \log_2(M_v)}, & \text{for } M-PSK \\ \frac{2q_v}{\log_2(M_v)}, & \text{for } M-QAM \end{cases} \]

(23)

and

\[ B_v = \begin{cases} \frac{g_{PSK,v} \gamma_t S}{R_v t_v N}, & \text{for } M-PSK \\ \frac{g_{QAM,v} \gamma_v S}{R_v t_v N}, & \text{for } M-QAM \end{cases}. \]

(24)

From this, it is shown in the Appendix (Derivation I) that the throughput-maximizing fractional power allocations \( \beta_v \) have to obey

\[ \beta_v = \left[ \sum_{u=1}^{K} \alpha_u \left( \frac{u_v - 1}{u_w} - \frac{1}{u_w} \right) \right] ^{-1} \]

(25)

where \( u_{max} = \max(u_1, \ldots, u_K) \).

C. Performance of Algorithm

The performance of the developed algorithm is assessed by means of Figs. 4–7 for M-QAM schemes only. Note that if reference is made to the nonoptimized scenario, then only the fractional transmission power is not meant to be optimized since the frame duration is easily related to the modulation order. Further note that the obtained graphs are generally labeled with parameter \( p \) that is defined as

\[ p = \Delta \left[ 10 \log_{10} \left( \frac{\gamma_1}{\gamma_1} \right), 10 \log_{10} \left( \frac{\gamma_2}{\gamma_1} \right), \ldots, 10 \log_{10} \left( \frac{\gamma_K}{\gamma_1} \right) \right] \]

which characterizes the relative strength in decibels of the \( K \) relaying stages with respect to the first stage.

Explicitly, Fig. 4 depicts the end-to-end BER versus the SNR in the first link in decibels for various two-stage sensor networks deploying the developed fractional power allocation strategy (25), which is also compared against a numerically obtained optimum allocation and a nonoptimized allocation. The first scenario, where \( r_{1,2} = r_{1,2} = 1, M_{1,2} = 4 \) (QPSK) and \( p = [0, 0] \) dB, is entirely symmetric, which leads to the same performance for all three allocation strategies. The second scenario is the same as the first, with the only difference that the channel in the second stage is now ten times stronger than in the first stage, i.e., \( p = [0, 10] \) dB. The subsequently created nonsymmetric scenario reveals a performance difference between the optimized (solid lines) and nonoptimized (dashed line) power allocations.
It can be observed that the optimum and developed allocation strategies yield the same performance for any of the depicted configurations. Furthermore, the gain of an optimized system over a nonoptimized system is highest for very asymmetric cases, i.e., for \( t_1 = 2, r_1 = 2, t_2 = 2, r_2 = 1, M_1 = 256, M_2 = 64, \) and \( p = [0, 10] \) dB. At a target end-to-end BER of \( 10^{-5} \), about 1 dB in power can be saved.

Fig. 5 is similar in nature to Fig. 4, with the only difference that a three-stage sensor network is scrutinized. Similar observations can be made for these scenarios, where gains of almost 4 dB can be observed. This corroborates the importance of the derived allocation strategy.

The throughput of a two-stage system is illustrated in Fig. 6, which utilizes the fractional resource-allocation strategies (13) and (25). The system deployed has the number of bits fixed to \( B = 100 \); furthermore, for all configurations, \( M_{1,2} = 4 \) (QPSK) and \( p = [0, 10] \) dB. It can be observed that in the region of low SNR, the developed allocation strategy performs worse than the optimum one. This is obvious, as the fractional frame durations have been derived, assuming that the SNR is large.

For most of the transitional region from zero throughput to maximum throughput, however, the derived allocations yield near-optimum throughput. In contrast, no optimization exhibits drastic losses in the transitional region. For example, given a scenario with \( t_{1,2} = r_{1,2} = 2 \) operating at a normalized throughput of 0.5 bits/s/Hz, a transmission power of 2 dB, which mounts to approximately 40%, is lost.

Observe also that the cases of full-rate STBC at each stage yield the same maximum throughput, whereas the case of the STBC with a rate of 3/4 has lower maximum throughput, notwithstanding the fact that it constitutes the strongest link. This is due to the limiting spectral efficiency of the STBC with a rate that is less than 1. Clearly, the strength of the
The precision of the fractional allocation algorithm for fixed modulation indexes allows performing a final numerical optimization at each relaying stage over all possible modulation indexes. The low complexity of (13) and (25) guarantees that such optimization comes at little additional computational power.

Such numerical optimization was performed for a two-stage sensor network with $p = [0, 10]$ dB and $t_{1,2} = r_{1,2} = 2$. Each stage could choose a modulation index belonging to the set $M_{1,2} = (2, 4, 16, 64, 256)$; this leads to 25 possible combinations, which are calculated in a fraction of a second. The performance gains in terms of decreased transmission power or increased throughput are clearly shown in Fig. 7, where the near-optimum adaptive modulation per stage is compared against various fixed combinations.

For example, if the sensor network was to operate at a normalized throughput of 2 bits/s/Hz, then the best but fixed modulation index consumes a transmission power of 20 dB. However, the optimum selection consumes only 15 dB, which yields a performance benefit of 33%.

V. Maximum Throughput for Partial Data Exchange

Partial or no data exchange at each relaying stage results in parallel MIMO channels, all of possibly different strengths. An example of such clustering process is shown in Fig. 8, with none of the relaying sensors per cooperative stage communicating with each other.

Here, the first stage spans two independent single-input–single-output (SISO) channels with average attenuations $\gamma_{1,1}$ and $\gamma_{1,2}$, respectively. Each of these channels causes independent BERs, which are denoted as $P_{1,1}$ and $P_{1,2}$, respectively. Similarly, the second stage spans two independent multiple-input–single-output (MISO) channels, where the first MISO channel consists of channels with average attenuations $\gamma_{2,1}$ and $\gamma_{2,3}$, and the second MISO channel consists of channels with average attenuations $\gamma_{2,2}$ and $\gamma_{2,4}$. Furthermore, assuming an error-free input into the second VAA relaying tier, the BERs at the output of the MISO channels are $P_{2,1}$ and $P_{2,2}$. Finally, the third stage spans a single MISO channel with a BER of $P_{3,1}$.

Note that the r-Ss belonging to the same stage need to transmit at the same rate; furthermore, they obviously need to know which part of the STBC to transmit, which could be negotiated at discovery or determined randomly. Furthermore, it is assumed that the synchronization among all the sensors that belong to the same relaying stage is perfect. This is clearly an idealistic assumption; however, the next analysis can serve as an upper bound on the performance of such networks. An analysis of the synchronization errors is presented in [46].

Obtaining the exact end-to-end BER is not trivial because an error in the first stage may propagate to the t-S; however, it may also be corrected in the next stage. Referring to Fig. 8, for example, it is assumed that the same information bit is erroneously received over the link denoted as (1, 1) and correctly over (1, 2). Then, the STBC formed by (2,1) and (2,3) has one erroneous and one correct information bit as its input. Assuming that $\gamma_{2,3} \gg \gamma_{2,1}$, then the error does not further propagate since it will be outweighed by the correct bit. Alternatively, if $\gamma_{2,3} \ll \gamma_{2,1}$, then there is a large likelihood that the error propagates. This creates dependences between the error events at each stage depending on the modulation scheme used, the prevailing channel statistics, the average channel attenuations, and the STBC chosen. The fairly complex interdependences become tractable under asymptotic conditions, as will be outlined later in the text.
Generally, it is desirable to develop an approach that decouples the error events at the respective stages. To this end, it is assumed that the system operates at low error rates or sufficiently high SNRs, which causes only one error event at a time in the entire sensor network. This is a realistic assumption because one would design a network that yields near-error-free communication.

Let us assume that an error occurs in link (1,1); however, (1,2) is error free. Then, the probability that the error propagates further is related to the strengths of channels (2,1) and (2,3). The elegant asymptotic error rate analysis developed in [52] and further extended to distributed STBCs in [54] shows that the probability of such error to propagate is asymptotically proportional to the strength (i.e., first-order channel statistics) of the STBC branch it departs from, which is (2,1) for one of the two MISO channels and (2,2) for the other one.

Therefore, the probability that an error that occurred in link (1,1) with probability \( P_{1,1} \) will propagate through the MISO channel spanned by (2,1) and (2,3) is given as \( P_{1,1} \cdot \gamma_{2,1}/(\gamma_{2,1} + \gamma_{2,3}) \), where the strength of the erroneous channel (2,1) is normalized by the total strength of both subchannels. To capture the probability that such an error propagates until it reaches the t-S, all the possible paths in the network and the original probability of error weighed with the ratios between the respective path gains have to be found.

Taking the preceding discussion into account and assuming that at high SNRs, only one such error will occur at any link, the end-to-end BER for the network depicted in Fig. 8 can be expressed as (26), shown at bottom of the page. This can be simplified to

\[
P_{b,e2e}(e) = \left[ \xi_{1,1} P_{1,1}(e) + \xi_{1,2} P_{1,2}(e) \right] + \left[ \xi_{2,1} P_{2,1}(e) + \xi_{2,2} P_{2,2}(e) + \xi_{3,1} P_{3,1}(e) \right] \quad (27)
\]

where \( \xi_{v,i} \) is the probability that an error occurring in link \((v,i)\) will propagate to t-S. This result is easily generalized to sensor networks of any size and any form of partial data exchange. To this end, remember that there are \( Q_v \) data exchanging clusters at the \( v \)th stage, each of which will yield an error probability of \( P_{v,1}(e) \). The asymptotic end-to-end BER is hence given as

\[
P_{b,e2e}(e) = \sum_{v=1}^{K} \sum_{i=1}^{Q_v} \xi_{v,i} P_{v,i}(e) \quad (28)
\]

where the probabilities \( \xi_{v,i} \) are determined by the specific network topology. The BERs \( P_{v,i}(e) \) can be found in (9) and any of the previously derived SERs with an appropriate number of transmit and receive antennas per cluster, as well as the prevailing channel conditions. The applicability of the derived end-to-end BER is assessed using Figs. 9 and 10.

Explicitly, Fig. 9 compares the numerically obtained and the derived end-to-end BER versus the SNR in the first link for a two-stage network, as depicted in Fig. 8, without the second stage. For all the simulations, QPSK has been used. The graphs are labeled on the respectively utilized channel gains. For low SNRs, it can be observed that the derived BER differs from the exact one; however, for an increasing SNR, both curves converge.

Fig. 10 compares the numerically obtained and the derived end-to-end BER versus the SNR in the first link for a three-stage network, as depicted in Fig. 8. The curves are again labeled on the channel gains. As clearly shown in Fig. 10, the derived end-to-end BER holds with high precision for a variety of different scenarios.

The derived end-to-end BERs in the form of (28) allow one to assign optimum fractional powers \( \beta_{v}^{e} \) such that together with the fractional frame durations \( \alpha_{v}^{e} \), near-optimum end-to-end throughput is achieved. The fractional frame durations are clearly independent of the channel statistics.
or the choice of clustering in the high-SNR mode; therefore, (13) holds true for \( \alpha_{v \in [1, K]} \). The fractional power allocations are derived as follows:

Without loss of generality, let us assume that all links obey Rayleigh fading and have a different channel gain. The error rates are then governed by (3), where \( u \) has to be replaced by the number of subchannels created in each of the \( Q_v \) clusters. The fractional power allocations are derived in the Appendix (Derivation II) as

\[
\beta_v = \left[ \frac{1}{\sum_{u=1}^{K} \alpha_w^t} \sum_{i=1}^{Q_v} \sum_{j \in i} \xi_{v,i,j}^{-1} K_{v,i,j}^{-1} A_{v}^{-1} B_{v,i,j} \right]^{-1}
\]

(29)

where the notation \( j \in i \) represents the \( j \)th subchannel belonging to the \( i \)th cluster. The partial expansion coefficients \( K_{v,i,j} \) in the \( v \)th stage for the \( i \)th cluster can be written as

\[
K_{v,i,j} = \prod_{j' \in i \setminus j}^{} \frac{\gamma_{v,j}}{\gamma_{v,j} - \gamma_{v,j'}}
\]

(30)

which has \( u_{v,i} \) multiplicative terms. The constant \( A_v \) is given by (23), whereas

\[
B_{v,i,j} = \begin{cases} \frac{g_{vPSK,v} \gamma_{v,i}}{R_v S} \frac{S}{N}, & \text{for } M-PSK \\ \frac{g_{vQAM,v} \gamma_{v,i}}{R_v S} \frac{S}{N}, & \text{for } M-QAM. \end{cases}
\]

(31)

The throughput is similarly obtained for the case of full cooperation and is hence not further illustrated here.

VI. CONCLUSION

The performance of distributed multistage sensor networks has been analyzed in terms of error rates, achievable throughput, and attainable power savings when compared to traditional nondistributed networks. It has been assumed that a source sensor communicates with a target processing unit or target sensor via relaying sensors, in which each sensor is attributed only one antenna element. Furthermore, spatially adjacent relaying sensors have been grouped into VAAs. The accomplished distributed MIMO relaying channels allow low-complexity STBCs to be deployed at each relaying stage. The performance of such sensor network topology has been analyzed in three steps.

First, the error rates of STBCs over Rayleigh and Nakagami flat-fading channels with arbitrary channel gains and fading statistics have been given.

Second, the error rates have then been utilized to derive throughput-maximizing fractional resource-allocation algorithms, assuming full data exchange among the sensors belonging to the same relaying stage. The fractional power and frame duration allocations that maximize the end-to-end data throughput or alternatively minimize the required transmission power have been determined.

Third, the analysis has then been extended to the case of no or partial data exchange at each relaying stage. Examples have demonstrated the advantageous applicability of the derived strategies, where transmit power savings of up to 40% have been observed. In addition, an asymptotic expression for the end-to-end error rates in the case of partial data exchange at each relaying stage has been obtained.

The transmission process is known to consume between 30%–50% of the power in a sensor node [64]. Transmission power savings of up to 40%, which were facilitated by the derived allocation algorithms, hence yield significant overall power savings.

Finally, it should be noted that the analysis exposed in this paper is equally applicable to general ad hoc-type networks with deployed space-time block coding. If complexity is not the limiting factor, then distributed space-time trellis codes can be deployed and analyzed similarly to STBCs.

APPENDIX

Derivation I: To prove (25), (22) is rearranged as

\[
P_{b,2e}(e) \leq \frac{A_1}{(1 + B_1 \beta_v)} \sum_{v=1}^{K} \frac{A_v}{(1 + B_v \beta_v)}^{u_v}
\]

(32)

\[
= \frac{A_1}{(1 + \alpha_1^{-1} B_1 \beta_v)}^{u_v} \sum_{v=2}^{K} \frac{A_v}{(1 + B_v \beta_v)}^{u_v}
\]

\[
= \frac{A_1}{(1 + \alpha_1^{-1} B_1 (1 - \alpha_1 \beta_v))^{u_v}} \sum_{v=2}^{K} \frac{A_v}{(1 + B_v \beta_v)}^{u_v}
\]

The constraints \( \sum_{v=1}^{K} \alpha_v \beta_v = 1 \) for a TDMA-based relaying system have been used [32]. Without loss of generality, the fractional power allocation \( \beta_K \) for the last relaying stage will be derived. To obtain the optimum fractional power allocations that yield a minimum end-to-end BER, (32) is differentiated \( K - 1 \) times along \( \beta_v \in (2, K) \).
The obtained $K - 1$ equations are equated to zero to arrive at
\[
\frac{u_1 A_1 \alpha_1' B_1}{(1 + \alpha_1' B_1 (1 - \sum_{v=2}^{K} \alpha_v' \beta_v'))^{u_1 + 1}} = \frac{u_2 A_2 B_2}{(1 + B_2 \beta_2')^{u_2 + 1}} \quad (33)
\]
\[= \frac{u_K A_K B_K}{(1 + B_K \beta_K')^{u_K + 1}}. \quad (34)
\]

For low target SERs, $B_v \beta_v' > 1$ for any $v \in (1, K)$, which allows rearranging (33) and (34) to
\[
\frac{\alpha_1' - u_1 A_1}{u_1 A_1} \left( 1 - \sum_{v=2}^{K} \frac{\alpha_v' \beta_v'}{u_v^2 A_v^2} \right) \approx \frac{B_{u_1}^2}{u_1 A_1} \left( \beta_{u_1} \right)^{u_1 + 1} (35)
\]
\[= \frac{B_{u_2}^2}{u_2 A_2} \left( \beta_{u_2} \right)^{u_2 + 1} \quad (36)
\]
\[= \frac{B_{u_K}^2}{u_K A_K} \left( \beta_{u_K} \right)^{u_K + 1} \quad (37)
\]

This set of equations is difficult to resolve in closed form wrt any of the $\beta_v'(1, K)$. To this end, the $u_{\text{max} + 1}$ th square root is taken from (35)–(37), where $u_{\text{max}} = \max(u_1, \ldots, u_K)$ per definition. The choice of $u_{\text{max}}$ is motivated by the fact that the error in approximating $(\beta_{v'})^y$ by $\beta_v'$ for $0 < \beta_v' < 1$ and $y \leq 1$ is smaller compared to the case when $y > 1$. Since such approximation is vital in further steps, it has to be made sure that the approximation error for the preceding equations is minimal. This justifies the choice of $u_{\text{max}}$, as it guarantees that $y = (u_v + 1)/(u_{\text{max} + 1}) \leq 1$ for any $v \in (1, K)$. Equations (35)–(37) can hence be recast into
\[
\alpha_1' \approx \left( \frac{B_{u_1}^2}{u_1 A_1} \right)^{1/(u_{\text{max} + 1})} \left( 1 - \sum_{v=2}^{K} \frac{\alpha_v' \beta_v'}{u_v^2 A_v^2} \right)^{1/(u_{\text{max} + 1})} \quad (38)
\]
\[= \left( \frac{B_{u_2}^2}{u_2 A_2} \right)^{1/(u_{\text{max} + 1})} \quad (39)
\]
\[= \left( \frac{B_{u_K}^2}{u_K A_K} \right)^{1/(u_{\text{max} + 1})} \quad (40)
\]

which are easily resolved in favor of $\beta_2', \ldots, \beta_K'$, i.e.,
\[
\beta_2' \approx \left( \frac{u_K A_K^{-1} B_{u_1}^2}{u_2 A_2^{-1} B_{u_2}^2} \right)^{1/(u_{\text{max} + 1})} \quad (41)
\]
\[= \left( \frac{u_K A_K^{-1} B_{u_K}^2}{u_K A_K^{-1} B_{u_K}^2} \right)^{1/(u_{\text{max} + 1})} \quad (42)
\]

where, when inserted into (38), yields
\[
\beta_K' \cdot \sum_{v=1}^{K} \alpha_v' \left( \frac{u_K^{-1} A_K^{-1} B_{u_1}^2}{u_v^{-1} A_v^{-1} B_{u_v}^2} \right)^{1/(u_{\text{max} + 1})} = 1 \quad (43)
\]

This concludes the proof.

Derivation II: To prove (29), recall that the asymptotic end-to-end BER is given as
\[
P_{b, e_{2e}}(e) = \sum_{v=1}^{K} \sum_{i=1}^{Q_v} \xi_{v,i} P_{v,i}(e) \quad (44)
\]

where the weights $\xi_{v,i}$ are derived, depending on the network topology. With reference to (3), the BER of the $i$th cluster in the $v$th stage can be expressed as
\[
P_{v,i}(e) = \sum_{j \in i} K_{v,i,j} \cdot P_{PSK/AM}(1, t_v, R_v, \gamma_{v,j}, S/N, M_v) \quad (45)
\]

where the expansion coefficients are given in (30). With reference to the analysis exposed in Section IV-B, it can be upper bounded as
\[
P_{v,i}(e) \leq \sum_{j \in i} \frac{K_{v,i,j} A_v}{1 + B_v(i,j) \beta_v'} \quad (46)
\]

where the coefficients $A_v$ and $B_v(i,j)$ are given by (23) and (31), respectively. Inserting (46) into (44) yields
\[
P_{b, e_{2e}}(e) \leq \sum_{v=1}^{K} \sum_{i=1}^{Q_v} \sum_{j \in i} \xi_{v,i} K_{v,i,j} A_v \quad (47)
\]

The same procedure is now followed as already outlined in Derivation I, where (32) is extended by the additional sums; furthermore, $A_v$ is replaced by $\xi_{v,i} K_{v,i,j} A_v$ and $u_v$ by 1, which finally yields (29).

REFERENCES


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