A Double-Stage Multiuser Detector With FFT-Based Equalization for Asynchronous CDMA Ultra-Wideband Communication Systems

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Abstract—A novel multiuser transmission and detection scheme for high-speed asynchronous Ultra-Wideband (UWB) communication systems is proposed. Firstly, a block-spreading code-division multiple-access (BS-CDMA) scheme with zero correlation window (ZCW) is designed for application in asynchronous UWB communication systems with dense frequency selective fading propagation. As a result, multiple access interference can be completely removed by maintaining the code orthogonality. Secondly, the UWB channel model is converted from a block Toeplitz matrix into block circulant matrix. Consequently, the inter-symbol interference is eliminated through the use of a Fast Fourier Transform (FFT) based minimum mean square error equalization. It is shown that the proposed multiuser transmission and detection scheme for UWB systems can deliver superior performance relative to the existing schemes, with low computation complexity.

Index Terms—Asynchronous UWB, block spreading, fast Fourier transform, zero correlation window.

I. INTRODUCTION

SINCE the US Federal Communications Commission (FCC) approved the limited use of ultra-wideband (UWB) technology [1], the UWB impulse radio (IR) has been a fast emerging technology and drawn considerable attention with its unique features, such as multipath resolution, material penetration and carrierless transmit fashion, etc. Basically, an UWB impulse radio defined by FCC refers to an electromagnetic waveform that has -10dB bandwidth no less than either 500MHz or 20% of its center frequency [1].

As the UWB technology can potentially provide a high data rate with low power spectrum density, an UWB based high speed communication standard, which is named High Rate Wireless Personal Area Networks (WPAN), is recently developed by the IEEE 802.15.3 task group [2]. The UWB based WPAN provides high data rate wireless ad hoc connectivity for fixed, portable and moving devices within Personal Operating Space (POS), which is generally within 10~20 meters. In WPAN, UWB technology is envisioned to provide high definition real-time video and audio distribution, file exchange among storage devices and cable replacement for home entertainment systems and/or small office/home office (SOHO) [3].

The UWB systems can be classified into two categories: multi-band (MB) and single-band (SB). In an MB-UWB system, the overall spectrum is divided into several sub-bands. Consequently, it has a smaller front-end processing bandwidth, low power consumption and higher flexibility in frequency allocation than a SB-UWB system.

Two types of MB-UWB schemes, based on single carrier (SC) [4] and multicarrier (MC) [5] transmission mode, are proposed in recent research publications. The SC-MB-UWB assigns one or more sub-bands to only one user in order to avoid the multiple access interference. The primary disadvantage of the pulsed SC-MB-UWB is the high implementation complexity. It is caused by a requirement for multiple RF chains in order to collect enough multipath energy. In addition, switching time requirement can be very stringent at both transmitter and receiver. Recently, some researchers [6] have addressed the complexity issue for SC-MB-UWB with jointly applying block transmission and channel equalization in the frequency domain.

For MC-MB-UWB systems, several schemes based on different multiplexing techniques are proposed. The MC-MB-UWB based on orthogonal frequency division multiplexing (MC-OFDM-UWB) assigns an unique carrier frequency to each user [5], [10], [11], [12]. It is noted that the OFDM based MC-MB-UWB not only retains the advantages of pulsed SC-MB-UWB scheme, but also demonstrates other attractive features. For example, it has the ability to efficiently capture multipath energy with a single RF chain, while being insensitive to group delay variations. However, OFDM would increase the transmitter complexity and also inherently bring a high peak-to-average power ratio, which results in excessive power generation to that allowed by the FCC regulation [1]. Furthermore, MC-OFDM-UWB is sensitive to frequency offsets, which can severely degrade the system performance.

Another type of MC-MB-UWB scheme is proposed based on CDMA technique and interpreted as MC-CDMA-UWB [13] and [14]. In MC-CDMA-UWB, the information symbol can be either spread in the time or frequency domain. The main advantages of this concept are a flexible frequency allocation and robustness against narrowband interference (NBI).

For the single-band impulse radio systems, it operates in a carrierless fashion and the pulse occupies a very wide spectrum. According to the spreading codes employed, recent research on single band UWB could be largely classified into
two categories: time-hopping based UWB (TH-UWB) [16], [15], [17], [18] and direct sequence spread spectrum based UWB (DS-UWB) [19], [20], [21], [22], [23]. The TH-UWB schemes distinguish users by allocating different time slots from frame to frame. DS-UWB based schemes assign each user a unique spreading sequence. For both multiple access schemes, the information symbols can be modulated with various modulation schemes, such as pulse amplitude modulation (PAM) and pulse shape modulation (PSM). However, TH-UWB is inevitable choice of spread spectrum when using pulse position modulation (PPM) data modulation of UWB pulses. Recent research [24] demonstrates that DS-CDMA will outperform TH-PPM in terms of the SNR performance and can support a larger number of users while providing a better of quality of service.

In this paper, the proposed scheme is investigated based on single band DS-UWB WPAN system with carrierless fashion, where multiple users are co-existing with ad-hoc network configuration. However, it is straightforward to extend it into multi-band DS-UWB systems.

Commercial UWB communications systems are required to operate in a multiple users environment, where multiple access interference (MAI) coming from coexisting users can severely degrade the system performance. In addition to MAI, a dense UWB multipath channel also induces inter-symbol interference (ISI). Furthermore, an ultra-thin and low power impulse of the UWB impulse radio comes at the cost of much more stringent timing tolerances [8]. In particular, the asynchronism of users, together with MAI and ISI, will destroy the spreading sequences orthogonality in the DS-CDMA based UWB systems and severely degrade the system capacity.

Consequently, the proposed UWB multiuser detection scheme addresses both MAI, ISI and computation complexity challenges based on a general asynchronous UWB system. The contribution of this paper is twofold.

Firstly, we propose an MAI-free detection for the asynchronous or quasi-synchronous block spreading CDMA UWB systems. The motivation of MAI-free reception design is not only to reduce both computation complexity and the need for channel estimation, but also to avoid the inherent near-far effect in DS-CDMA systems. In previous research, a chip-interleaved block-spreading CDMA (CIBS-CDMA) system [25] and multistage block-spreading (MS-BS) PPM system [16] are proposed to achieve MAI-free reception. However, both CIBS-CDMA and MS-BS PPM work only in the synchronous system.

In this paper, the MAI-free reception in the asynchronous multiuser UWB communication systems is achieved by keeping the orthogonality of the spreading sequences with the incorporation of block spreading at the transmitter and double parallel despreading stages at the receiver. To keep the code orthogonality in asynchronous UWB system, a set of spreading sequences with a zero correlation window (ZCW) [29] or a zero correlation zone (ZCZ) [30] is used. The binary ZCW spreading sequences introduced in [30] are available only for even numbers of users. For odd numbers of users, binary ZCW sequences can only be achieved by padding zeros into the spreading sequences [29]. This, however, would cause a bandwidth expansion. In this paper, a set of analog ZCW spreading codes for asynchronous UWB systems is developed to maintain code orthogonality, with no bandwidth expansion compared to binary ZCW codes.

Secondly, a low complexity Fast Fourier Transform (FFT) based channel equalizer is proposed to eliminate ISI. Although a minimum mean square error (MMSE) UWB receiver [19] has demonstrated a superior performance in suppressing interference compared to a RAKE receiver, a large number of multipath in an UWB channel makes it impractical of directly extension from the narrowband/wideband systems to UWB systems. The proposed equalization method is based on representing the UWB channel of each user as a block circulant matrix. The proposed receiver achieves a better performance and much lower complexity than the conventional MMSE receiver [19].

The paper is organized as follows. The system and signal models are described in Section II. Section III considers a scheme to completely remove MAI with the proposed ZCW spreading sequences design for asynchronous systems. In Section IV, an FFT-based MMSE channel equalization strategy is proposed. In Section V, the signal to interference and noise ratio of the proposed scheme is derived in a close form and the corresponding simulation results are also provided to validate the analysis. Furthermore, the computation complexity is investigated and compared with some best known systems. Finally, the conclusions are given in Section VI.

Notation: Bold upper (lower) letters denote matrices (column/raw vectors); $(\cdot)^\dagger$, $(\cdot)^T$ and $(\cdot)^H$ denote the conjugate, transpose and Hermitian transpose, respectively; $I_k$ denotes the identity matrix of size $k$; $0_{k \times p}$ denotes an all-zero matrix with size $k \times p$ and $0_k$ represents an all-zero square matrix with size $k$. $\lfloor a \rfloor$ stands for the largest integer not bigger than $a$ and $\otimes$ denotes the Kronecker product.

II. SYSTEM DESCRIPTION

The block diagram in Fig. 1 models the proposed asynchronous UWB communication system with $K$ users. In order to simplify the figure, only the receiver for user $k$ is shown.

A. Transmitter Structure

For each user, the binary data stream can be divided into data blocks, each of which consists of $N_c$ symbols. The data
block for user $k$, $k \in \{1, \cdots, K\}$ is represented by a vector, denoted by $b_k$, and given by

$$b_k = (b_k(i))^T, i \in \{1, \cdots, N_c\},$$  

(1)

where $b_k(i) \in \{-1, 1\}$.

When $T_s$ denotes the symbol duration, the period of the data block is $N_c T_s$. The data block is then modulated with binary pulse amplitude modulation (BPAM) by a ultra-thin monopulse, denoted by $\omega(t)$. That is, $b_k(i) = 1$ is mapped to $\omega(t)$ and $b_k(i) = -1$ is mapped to $-\omega(t)$. According to a typical UWB pulse in the literature [7], the transmitted waveform is shaped as the second derivative of the Gaussian function and given by

$$\omega(t) = \sqrt{P} \left[ 1 - 4\pi \left( \frac{t}{\tau_m} \right) \right] e^{2\pi \left( \frac{t}{\tau_m} \right)^2},$$  

(2)

where $P$ is the transmit power and $\tau_m$ denotes a time normalization factor.

At the transmitter side, direct-sequence spread-spectrum (DSSS) modulation is employed. The DSSS signal to be transmitted for the $k$-th user is denoted by $y_k(t)$ and obtained as

$$y_k(t) = \sum_{n_c=1}^{N_f N_c} x_k(n_c) \cdot \omega(t - n_c T_s),$$  

(3)

where $x_k(n_c)$ is the $n_c$-th element of vector $x_k$, which is given by

$$x_k = s_k \otimes b_k,$$  

(4)

where $s_k = [s_{k1}, s_{k2}, \cdots, s_{kN_f}]^T$ is the spreading sequence for user $k$ and $N_f$ is the spreading gain. It is obvious that $x_k$ is an $N_f N_c$ sized vector.

B. Channel Model

The channel impulse response model [28], in which the multiple paths (rays) arrives in form of cluster, is given by

$$h(t) = \sum_{l_c=1}^{L_c} \sum_{l_r,l_c} \alpha_{(l_c,l_r,l_c)} \delta(t - T_{l_c} - \tau_{(l_c,l_r,l_c)}),$$  

(5)

where $L_c$ is the number of clusters, $L_{r,l_c}$ represents the number of multipath in the $l_c$-th cluster, $\alpha_{(l_c,l_r,l_c)}$ is the multipath gain coefficient of the $l_r,l_c$-th path within the $l_c$-th cluster and can be defined as

$$\alpha_{(l_c,l_r,l_c)} = p_{(l_c,l_r,l_c)} \beta_{(l_c,l_r,l_c)},$$  

(6)

where $p_{(l_c,l_r,l_c)}$ is equally likely to take on the values of $\pm 1$ and $\beta_{(l_c,l_r,l_c)}$ is the lognormal fading term, defined as

$$E[\beta_{(l_c,l_r,l_c)}] = \Omega \sigma e^{-\frac{\gamma}{2}} e^{-\frac{\tau_{(l_c,l_r,l_c)}}{\tau_c}},$$  

(7)

where $\Gamma$ is the cluster decay factor, $\gamma$ is the multipath decay factor, $T_{l_c}$ is the arrival time of the first path of the $l_c$-th cluster and $\tau_{(l_c,l_r,l_c)}$ is the delay of the $l_r,l_c$-th ray within the $l_c$-th cluster relative to the first path arrival time, $T_{l_c}$. The distribution of the cluster arrival time and the ray arrival time are given [28] by

$$p(l_c) = \Lambda \exp(-\Lambda(T_c - T_{(l_c-1)})),$$  

$$p(l_c, \tau_{r,l_c}) = \zeta \exp\left[-\zeta \left(\tau_{(l_c,l_r,l_c)} - \tau_{(l_c,l_r,l_c-1)}\right)\right],$$  

(8)

where $\Lambda$ is the cluster arrival rate and $\zeta$ is the multipath arrival rate within each cluster.

Consequently, the sampled discrete-time equivalent channel finite impulse response (FIR) corresponding to the $k$-th user can be expressed in a vector form as

$$h_k = [h_{k1}, h_{k2}, \cdots, h_{kN_{m,k}}]^T,$$  

(9)

in which $h_{ki}$, $i \in \{1, \cdots, N_{m,k}\}$ is the multipath gain and $N_{m,k} = \left\lfloor \frac{T_{m,k}}{T_r} \right\rfloor$ is the number of resolvable multipath within the maximum channel delay spread, where $T_{m,k}$ denotes the maximum delay spread of the multipath channel for user $k$ and $T_r$ is the resolution of the multipath channel.

C. Receiver Structure

With a quasi-static multipath UWB channel model, of which the channel state information keeps constant for consecutive $N_f N_c$ symbol periods, the output of the receive antenna is denoted by $r(t)$ and obtained as

$$r(t) = \sum_{k=1}^{K} h_k(t) x_k(t) + n(t),$$  

(10)

where $h_k(t)$ is the channel impulse response for user $k$ and $n(t)$ is additive Gaussian distributed noise with zero mean and variance of $\sigma^2$. Consequently, the sampled output of the matched filter is denoted by a vector $r$ and given as

$$r = \sum_{k=1}^{K} \begin{pmatrix} 0_{N_{rc} \times N_c N_f} \\ H_{k,N_f} \end{pmatrix} \left( s_k \otimes b_k \right) + n,$$  

(11)

where $n$ denotes an additive white Gaussian noise (AWGN) vector with each element being a zero mean variable with variance $\sigma^2$; $N_{rc} = \left\lfloor \frac{T_r}{T_c} \right\rfloor$ is the number of resolvable multipath within delay $\tau_c$, which denotes the arrival timing delay of the $k$-th user compared to the first user; Without loss of generality, we assume that $\tau_c > \tau_{(K-1)} \geq \cdots \geq \tau_1 = 0$; $N_{(r_{\max} - \tau_c)} = N_{(r_{\max} - \tau_c)}$ is the number of resolvable multipath within time duration $(\tau_{K} - \tau_c)$. The users’ asynchronism mainly comes from timing jitter [9], propagation delay and less accuracy of the time references [16].

The channel model $H_{k,N_f}$ is a block-Toeplitz matrix, which can be represented as

$$H_{k,N_f} =$$

\[ \text{Diagram} \]

\[ \text{Equation} \]
in which \( N_r = \left\lfloor \frac{\tau_{\text{max}}}{T_c} \right\rfloor \) denotes the number of resolvable multipath within a symbol duration \( T_s \). The matrix \( H_{k,m} \) consists of \( N_c N_f \) channel impulse response vectors \( h_k \) defined in Eq. (9).

The received sequence \( r \) in Eq. (11) can be illustrated in Fig. 2, in which the shaded block represents the inter data block chip interference, which is caused by the multipath spread coming from the previous data block chip.

In order to simplify the analysis, the time scale, starting from the first user’s signal arrival time, can be divided into \( N_f + 1 \) frames as shown in Fig. 2. Frames (1) to \( (N_f) \) consist of \( N_c N_f \) samples of each and Frame \( (N_f+1) \) contains \( N_f k + N_{\text{max}}^{m,k} - N_r \) samples, where \( N_{\text{max}}^{m,k} \) is defined as

\[
N_{\text{max}}^{m,k} = \max\{N_{m,k}, k \in \{1, \cdots, K\}\}
\]

and \( (N_{\text{max}}^{m,k} - N_r) \) represents the maximum number of the samples in the shaded block.

By combining Frame \( (N_f+1) \) and Frame \( (1) \) in Fig. 2, the received sequence can be depicted as in Fig. 3.

This procedure will enhance the noise power spectral density from \( \sigma^2 \) into \( \sigma^2(1 + \frac{N_c N_r - N_f k + N_{\text{max}}^{m,k} - N_f r}{N_f N_c N_r}) \). As the duration of data block is chosen to be much larger than the multipath spreading delay and transmission delay, or \( N_f N_c N_r \gg N_f k + N_{\text{max}}^{m,k} - N_r \), the noise power enhancement can be neglected.

By defining the Toeplitz channel matrix \( H_k \) as

\[
H_k = \begin{bmatrix}
    h_k & h_{k+1} & \cdots & h_{k+N_c-1} \\
    h_{k+N_c} & h_{k+N_c+1} & \cdots & h_{k+2N_c-1} \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{k+(N_c-1)N_f} & h_{k+(N_c-1)N_f+1} & \cdots & h_{k+2(N_c-1)N_f-1}
\end{bmatrix}
\]

(14)

where \( H_{k1} \) and \( H_{k2} \) are the sub-matrices of the block Toeplitz channel matrix \( H_k \) and is given by

\[
H_{k1} = H_{k} (1 : (N_c N_r - N_f k), :)
\]

\[
H_{k2} = H_{k} ((N_c N_r - N_f k + 1) : (N_c N_r - N_r + N_{m,k}), :)
\]

The modified received sequence shown in Fig. 3 is denoted by \( N_f N_c N_r \)-sized vector \( \bar{r} \) and given by

\[
\bar{r} = \sum_{k=1}^{K} \left( I_{N_f} \otimes H_{k1} \right) (s_k \otimes b_k)
\]

\[
+ \sum_{k=1}^{K} \left( I_{N_f} \otimes H_{k2} \right) (\bar{s}_k \otimes b_k) + \bar{n}
\]

\[
= \sum_{k=1}^{K} \left( s_k \otimes (H_{k1} b_k) + \bar{s}_k \otimes (H_{k2} b_k) \right) + \bar{n}
\]

(15)

in which \( s_k = [s_k N_f, s_1, s_2, \cdots, s_{k(N_f-1)}] \) is one chip shifted version of \( s_k \) to the left and matrices \( H_{k1} \) and \( H_{k2} \) are defined by

\[
H_{k1} = \begin{bmatrix}
    0_{N_r \times N_c} & H_{k1}
\end{bmatrix}
\]

\[
H_{k2} = \begin{bmatrix}
    H_{k2} & 0_{((N_c-1)N_r - N_{m,k}-N_{r_k}+1) \times N_c}
\end{bmatrix}
\]

(16)

are \( N_c N_r \times N_c \) matrices and \( \bar{n} \) is the corresponding AWGN vector.

In Eq. (15), \( \sum_{k=1}^{K} \bar{s}_k \otimes (H_{k2} b_k) \) is the interference due to the time delay \( T_k \), \( k \in \{1, \cdots, K\} \).

III. MAI CANCELLER FOR ASYNCHRONOUS BS-CDMA UWB SYSTEMS

Block-spreading CDMA multiuser detection schemes [16], [25] are well known for the property to completely eliminate the multiple access interference from other users after frequency selective propagation. The recent published BS-CDMA schemes are proposed for synchronous communication systems. However, the synchronization is not a realistic assumption for high speed UWB system due to the ultra-thin and low power pulses. As a result, the orthogonality of the spreading sequences is destroyed at the receiver in asynchronous systems and BS-CDMA will consequently not be able to completely eliminate MAI.

In this section, a novel block spreading CDMA based strategy is developed to achieve MAI-free reception in asynchronous UWB systems.

The modified received sequence \( \bar{r} \) in Eq. (15) is serial to parallel converted to form a \( N_f \times N_c N_r \) matrix, which is denoted by \( \bar{R} \) and given by

\[
\bar{R} = \begin{bmatrix}
    \bar{r}_1^T & \bar{r}_2^T & \cdots & \bar{r}_{N_f}^T
\end{bmatrix}^T
\]

(17)
in which
\[
\bar{\mathbf{R}}_i = [\bar{\mathbf{R}}((i-1) N_c N_r + 1), \ldots , \bar{\mathbf{R}}(i N_c N_r)] ,
\]
i \in \{1, \ldots , N_f \}.

In order to remove MAI in Eq. (15), the spreading codes is specified as:
\[
\mathbf{s}_i = (\mathbf{s}_i | \mathbf{s}_i \mathbf{r}_j^T = N_f \delta(i-j) ; \mathbf{s}_i \mathbf{r}_j^T = 0, \forall i, j \in \{1, \ldots , K\}.
\]

Consequently, for the \(k\)-th user, the modified data block \(\bar{\mathbf{R}}\)

in Eq. (17) is de-spread in two parallel stages by \((\mathbf{s}_k)^T\) and \((\mathbf{s}_k \mathbf{H}_1)^T\) as depicted in Fig. 4.

The output of the first stage despreading, denoted by a \(N_c N_f\)-sized vector \(\mathbf{d}_{1k}\), is given by
\[
\mathbf{d}_{1k} = \mathbf{s}_k^T \bar{\mathbf{R}} + \sum_{i=1}^{K} (\mathbf{s}_i \mathbf{s}_i^T) \mathbf{H}_1 \mathbf{b}_i + \mathbf{s}_k^T \mathbf{n} = \mathbf{H}_{k1} \mathbf{b}_k + \mathbf{s}_k^T \mathbf{n} = \left(0_{N_{rXY} \times N_c} \mathbf{H}_{k1}\right) \mathbf{b}_k + \mathbf{n}_{1k}.
\]

The output of the second stage despreading, denoted by a \(N_c N_f\)-sized vector \(\mathbf{d}_{2k}\), is given by
\[
\mathbf{d}_{2k} = \mathbf{H}_{k2} \mathbf{b}_k + \mathbf{s}_k \mathbf{n} = \left(0_{((N_c+1) N_r - N_{m,k} - N_{r_k} + 1) \times N_c} \mathbf{H}_{k2}\right) \mathbf{b}_k + \mathbf{n}_{2k},
\]

where \(\mathbf{n}_{1k}\) and \(\mathbf{n}_{2k}\) are the corresponding additive noise vectors.

The detector for user \(k\) constructs a new detected data vector by selecting the last \((N_c N_r - N_{r_k})\) elements in the vector \(\mathbf{d}_{1k}\) and the first \((N_{m,k} + N_{r_k} - N_r)\) elements in the vector \(\mathbf{d}_{2k}\). Consequently, the detected data block after two parallel despreading stages for the \(k\)-th user, denoted by \(\mathbf{d}_k\), can be represented as
\[
\mathbf{d}_k = \left(\mathbf{H}_{k1}\right) \mathbf{b}_k + \mathbf{n}_{1k} = \mathbf{H}_{k2} \mathbf{b}_k + \mathbf{n}_{2k},
\]

where \(\mathbf{n}_k\) is the corresponding additive noise vector with the last \((N_c N_r - N_{r_k})\) elements in the vector \(\mathbf{n}_{1k}\) and the first \((N_{m,k} + N_{r_k} - N_r)\) elements in the vector \(\mathbf{n}_{2k}\). It is obvious from Eq. (22) that the MAI of user \(k\) has been removed.

The spreading sequences proposed in Eq. (19) is a set of sequences with zero correlation window of 3 [30]. A set of sequences with ZCW can achieve both zero auto-correlation sidelobe and zero cross-correlation function within the width of zero correlation window. The sequence set with ZCW of 3 means that when users’ maximum asynchronous shift is within \([-N_r T_{s,k} + N_{r_k} T_{s,k}]\), the orthogonality of the spreading sequences can be kept at the receiver.

Although a large ZCW or a large zero correlation sidelobe means that large multiuser asynchronism can be tolerated, this would be at the expense of a reduced bandwidth efficiency and a more complex generation procedure [30]. As the multipath delay spread in UWB systems can be in excess of hundreds of symbol periods, MAI-free reception in conventional DS-CDMA systems can only be achieved by adopting the spreading sequences with a ZCW width as large as several hundred chips, which is practically impossible.

The main advantage of the proposed block-spreading CDMA compared to the conventional DS-CDMA in UWB systems is that the MAI-free reception can be achieved with small ZCW spreading sequences.

In order to reduce the ZCW width as small as 3, the block size \(N_c\) was chosen to meet the condition \((N_c+1) T_{s} > \tau_K + T_{m,k}\). In other words, a ZCW of 3 is applicable only when data block inter-chip interference is only coming from the adjacent data block chip.

For the proposed scheme with an even number of users, the binary sequences with ZCW of 3 are investigated in [30]. For the odd number of users, there are no existing binary spreading sequences that can achieve the bandwidth efficiency as same as for an even number users. As a result, we propose an analog spreading code set generated by computer searching, which is demonstrated in Appendix (1). It is shown that the proposed analog spreading code set has ZCW of 3 with satisfying the design constraint in Eq. (19) and maintains the same bandwidth efficiency as the binary codes in [30].

IV. FFT-BASED CHANNEL EQUALIZATION FUNCTION

As MAI has been completely removed by the two-stage MAI canceller as shown in Eq. (22), a channel equalizer is proposed in this section to eliminate inter-symbol interference for the UWB systems. Due to the large number of multipath in UWB channel impulse response, the conventional equalization schemes, for example, zero forcing and minimum mean square error [32], implemented in a standard manner would be very complex. In this section, an FFT-based MMSE channel equalization scheme with low computation complexity is proposed.
In the FFT-based MMSE equalizer depicted in Fig. 4, an MAI-free sequence at the output of the MAI canceller, \( d_k \) from Eq. (22), is first multiplied by a matrix \( H_{\text{tran}} \), given by

\[
H_{\text{tran}} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

where \( \delta' \) is a \((N_c, N_r - (N_m,k) - N_r)) \times (N_m,k - N_r)\) zero matrix and \( H_{\text{tran}} \) is \(N_c N_r \times ((N_c - 1) N_r + N_m,k)\) matrix.

The resulting sequence, denoted by \( d_{\text{tran}}^k \), is given by

\[
d_{\text{tran}}^k = H_{\text{tran}} d_k = H_{\text{tran}} (H_k b_k + n_k)
\]

where \( H_k \) is the equivalent block-circulant channel matrix given by

\[
\tilde{H}_k = [c_{00}, c_{01}, \ldots, c_{(N_c-1)}]
\]

where each column is obtained by a cyclic shift of the previous column down by \( N_c \) chips. Consequently, we define that \( c_{ei} \) is the circular shift of \( c_{e0} \) by delay \( i N_c \) chips, which is called circular shift delay. \( n_k \) is an additive noise vector. This procedure is equivalent to overlap the last \( N_m,k-N_r \) rows of \( H_k \) with its first \( N_m,k-N_r \) rows. It is obvious that the pre-multiplied matrix \( H_{\text{tran}} \) is to convert the channel matrix from a block-Toeplitz matrix into a block-circulant matrix. By doing so, the first \( N_m,k-N_r \) elements of \( N_c N_r \)-sized noise vector \( n_k \) is colored by matrix \( H_{\text{tran}} \). However, the effect of \( H_{\text{tran}} \) on \( n_k \) is diminished when \( N_c N_r \gg (N_m,k-N_r) \).

The equivalent block-circulant channel matrix \( \tilde{H}_k \) shown in Eq. (25) can be decomposed by a Fast Fourier Transform [26], [27] as

\[
\tilde{H}_k = F_{(N_c,N_r)}^H \Delta_k F_{N_c},
\]

in which \( F_{(N_c,N_r)} = F_{N_c} \otimes I_{N_r} \), where \( F_{N_c} \) stands for an \((N_c \times N_c)\)-sized Fourier matrix and is defined as

\[
[F_{N_c}]_{i,l} = N_c^{-1/2} e^{(-j2\pi(i-1)(l-1)/N_c)}
\]

\[(i,l) \in \{1, \ldots, N_c\}; \]

\( \Delta_k \) is a block diagonal matrix given as

\[
\Delta_k^H = \begin{pmatrix}
\delta_{11} & \cdots & 0 & 0 \\
0 & \delta_{21} & \cdots & 0 \\
& \ddots & \ddots & \ddots \\
0 & 0 & \cdots & \delta_{N_c,1} \cdots \delta_{N_c,N_c}
\end{pmatrix}
\]

where \( \delta_{ni}, n \in \{1, \ldots, N_c\} \) and \( i \in \{1, \ldots, N_r\} \), is a real number. It is important to note that

\[
\Delta_k^H \Delta_k = \text{diag}(\lambda_{k1}, \lambda_{k2}, \ldots, \lambda_{kN_r}),
\]

where \( \lambda_{kn} = \sum_{n=1}^{N_r} |\delta_{ni}|^2, n \in \{1, \ldots, N_c\} \), is the eigenvalue of Hermitian matrix \( H_k^H H_k \).

The MMSE equalizer taps for user \( k \), denoted by \( \Gamma_k^{\text{MMSE}} \), are given by [32]

\[
\Gamma_k^{\text{MMSE}} = \arg \min_{r_k \in C^{N_c \times N_r}} \mathbb{E}\{\|b_k - \Gamma_k (H_k b_k + n_k)\|^2\}.
\]

The solution to Eq. (29) can be expressed as [32]

\[
\Gamma_k^{\text{MMSE}} = H_k^H (PH_k H_k^H + \sigma_e^2 I_{N_c,N_r})^{-1};
\]

where \( P \) is the transmission power per symbol and \( \sigma_e^2 \) is the power spectrum density of the additive noise. By substituting Eq. (26) into Eq. (30), we get

\[
\Gamma_k^{\text{MMSE}} = F_{N_r}^H \Delta_k^H F_{(N_c,N_r)} (PF_{(N_c,N_r)}^H \Delta_k F_{N_c} F_{N_r}^H \Delta_k^H F_{(N_c,N_r)} + \sigma_e^2 I_{N_c,N_r})^{-1}
\]

By applying the FFT-based block circulant matrix decomposition, the MMSE equalization scheme becomes easy to implement by inverting an \(N_c \times N_c\) diagonal matrix \((\Delta_k^H \Delta_k + \sigma_e^2 I_{N_c})\) in Eq. (31), instead of inverting an \((N_c N_r) \times (N_c N_r)\) square matrix \((H_k H_k^H + \sigma_e^2 I_{N_c,N_r})\) in Eq. (30) as a standard implementation. For the UWB systems with a large channel matrix \( H_k \), the FFT-based MMSE equalization results in a significant computation complexity reduction.

Consequently, the output of the FFT-based equalizer is denoted by \( \hat{b} \) and obtained as

\[
\hat{b}_k = \Gamma_k^{\text{MMSE}} (H_k b_k + \tilde{n}_k)
\]

\[
= F_{N_r}^H (P \Delta_k^H \Delta_k + \sigma_e^2 I_{N_c})^{-1} \Delta_k^H \Delta_k F_{N_c} b_k^T
\]

\[
+ F_{N_r}^H (P \Delta_k^H \Delta_k + \sigma_e^2 I_{N_c})^{-1} \Delta_k^H F_{(N_c,N_r)} \tilde{n}_k
\]

\[
= F_{N_r}^H \Phi_k F_{N_c} b_k + F_{N_r}^H \Psi_k \tilde{n}_k',
\]

where an element of \( \tilde{n}_k' \), given by \( F_{(N_c,N_r)}^H \tilde{n}_k \), can be roughly approximated as a zero mean Gaussian distributed variable with variance \( \sigma_e^2 \). \( \Phi_k \) is defined as

\[
\Phi_k = \begin{pmatrix}
\frac{\lambda_{k1}}{P_{k1} + \sigma_e^2} & \cdots & 0 \\
0 & \frac{\lambda_{k2}}{P_{k2} + \sigma_e^2} & \cdots & 0 \\
& \ddots & \ddots & \ddots \\
0 & 0 & \cdots & \frac{\lambda_{kN_r}}{P_{kN_r} + \sigma_e^2}
\end{pmatrix}
\]

and

\[
\Psi_k = \begin{pmatrix}
\frac{\sqrt{\lambda_{k1}}}{P_{k1} + \sigma_e^2} & 0 & \cdots & 0 \\
0 & \frac{\sqrt{\lambda_{k2}}}{P_{k2} + \sigma_e^2} & \cdots & 0 \\
& \ddots & \ddots & \ddots \\
0 & 0 & \cdots & \frac{\sqrt{\lambda_{kN_r}}}{P_{kN_r} + \sigma_e^2}
\end{pmatrix}
\]
V. PERFORMANCE ANALYSIS

In this section, the signal to interference and noise ratio (SINR) of the proposed scheme is derived. Furthermore, its computation complexity is calculated. Finally, the simulation results are provided along the comparison with some best known UWB multiuser detection strategies.

A. Signal to Interference and Noise Ratio

For the FFT-based MMSE equalizer, the power of additive noise corresponding to the i-th symbol of the k-th user’s data block can be obtained from Eq. (32), as

$$P_{awgn}^i = \frac{\sigma^2}{N_c} \sum_{j=1}^{N_c} \frac{\lambda_{kj}}{(P\lambda_{kj} + \sigma_c^2)^2}.$$  \hspace{1cm} (35)

Similarly, the power of the i-th symbol and of its corresponding ISI are, respectively, given as

$$P^i = \frac{P}{N_c} \left( \sum_{j=1}^{N_c} \frac{\lambda_{kj}}{(P\lambda_{kj} + \sigma_c^2)} \right)^2;$$

$$P_{ISI}^i = \frac{P}{N_c} \sum_{j=1}^{N_c} \frac{\lambda_{kj}^2}{(P\lambda_{kj} + \sigma_c^2)} - \frac{P}{N_c} \left( \sum_{j=1}^{N_c} \frac{\lambda_{kj}}{(P\lambda_{kj} + \sigma_c^2)} \right)^2.$$  \hspace{1cm} (36)

Thus, the corresponding SINR for the i-th symbol of the k-th user is given in Eq. (37), which is found at the top of the next page.

Eq. (37) indicates that the symbols within the same data block will have the same SINR at the receiver.

For the proposed scheme, we have

$$SINR_{ik}^{MMSE} = \frac{\sum_{j=1}^{N_c} \frac{P\lambda_{kj}}{(P\lambda_{kj} + \sigma_c^2)}}{\sum_{j=1}^{N_c} \frac{\sigma^2}{(P\lambda_{kj} + \sigma_c^2)}} \leq G_H \frac{P}{\sigma_c},$$  \hspace{1cm} (38)

where $P/\sigma_c$ is the SNR of user k in an AWGN channel and $G_H$ represents the percentage of the captured energy relative to the total energy in all multipaths. In Eq. (38), the equality is achieved when $\lambda_{k1} = \lambda_{k2} = \cdots = \lambda_{kN_c} = G_H$, where $\lambda_{ki}$ represents an eigenvalue of Hermitian matrix $H_k^H H_k$, given as

$$H_k^H H_k = \begin{pmatrix} \rho_{k0} & \rho_{k1} & \cdots & \rho_{kN_c} \\ \rho_{k1} & \rho_{k0} & \cdots & \rho_{kN_c} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{kN_c} & \rho_{kN_c} & \cdots & \rho_{k0} \end{pmatrix}.$$  \hspace{1cm} (39)

The derivation of the SINR bound in Eq. (38) is given in Appendix (II).

As $H_k^H H_k$ is a circulant matrix, its eigenvalue $\lambda_{km}$, $m \in \{1, \cdots, N_c\}$, can be represented as

$$\lambda_{km} = \sum_{i=0}^{N_c-1} \rho_{ki} e^{2\pi im/N_c} = G_H + \sum_{i=1}^{N_c-1} \rho_{ki} e^{2\pi im/N_c},$$  \hspace{1cm} (40)

where $\rho_{ki}$ denotes the correlation between the first column and the $(i+1)$-th column of the equivalent channel matrix $H_k$ and is given by

$$\rho_{ki} = c_{ki}^T c_{ki}, i \in \{0, \cdots, N_c - 1\}.$$  \hspace{1cm} (41)

Fig. 5 demonstrates the relation of the correlation $\rho_{ki}$ and circular shift delay, where $N_c = 150$ and LOS UWB channel impulse response model is characterized in Table I.

<p>| Table I: Parameters of Channel Impulse Responses in LOS and NLOS UWB Channel |</p>
<table>
<thead>
<tr>
<th>Parameters</th>
<th>LOS</th>
<th>NLOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster Arrival Rate (1/nsec)</td>
<td>1/60</td>
<td>1/11</td>
</tr>
<tr>
<td>Ray arrival rate (1/nsec)</td>
<td>1/0.5</td>
<td>1/0.35</td>
</tr>
<tr>
<td>Cluster decay factor</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Ray decay factor</td>
<td>8.5</td>
<td>8.5</td>
</tr>
<tr>
<td>Revolution $T_r$ (nsec)</td>
<td>0.167</td>
<td>0.167</td>
</tr>
</tbody>
</table>

Fig. 5. Example of the relation between $P_{awgn}$ and circular shift delay, where $N_c = 150$ and LOS UWB channel impulse response model is characterized in Table I.

B. Simulation Results

In this section, we present the simulation results for the performance of the proposed multiuser detection scheme for asynchronous UWB communication systems. In simulations, the channel impulse response model proposed by Intel [28] is used. The channel models for different users are independently generated. The characteristic of the line-of-sight (LOS) and nonline-of-sight (NLOS) UWB channel models used in this paper are shown in Table I and illustrated in Fig. 6.

In the simulations, equal transmit power for all users is assumed and only the information of channel impulse response associated with interested user is known at the corresponding receiver. In the other word, we do not assume that the receiver
has all users’ channel state information (CSI). The CSI of the desired user can be estimated by periodically inserting the pilot symbols among the information symbol stream. The channel estimation error reaches its minimum when the pilot insertion frequency is greater or equal to two times the maximum Doppler spread of the fading channel. As most of the high data rate UWB devices are going to be deployed in WPAN, it is reasonable to assume the channel fading rate in time domain is low. Thus, the channel can be sampled at a sufficient rate. Consequently, there is little degradation due to the channel estimation errors.

Furthermore, We also assume that in our simulations, the energy from all multipath can be collected, so that the BER performance in AWGN channel can be used as a lower bound to evaluate the performance of the proposed scheme. This assumption can be satisfied once the sampling rate is high enough so that all resolvable multipath can be detected at the receiver, although the high sampling rate could be prohibited in practical systems due to the high power consumption and the high complexity for implementation. It is obvious that a relatively low sample rate will decrease the amount of the collected power and simplify the implementation. However, it will not affect the superior performance of the proposed scheme to other best known proposals, as when the sampling rate is reduced, the BER performances associated with various schemes will all get degraded according to the degree of power loss. In all simulations, the BPAM is selected as the modulation scheme and the time delay $\tau_k$ for user $k$, $k \in \{1, \cdots, K\}$ is an uniformly distributed variable between 0ns and 60ns.

Fig. 7 demonstrates the impact of the transmission rate on the bit error rate (BER) performance. It is shown that the BER performance approaches the AWGN bound as the transmission rate reduces. When the transmission rate is 10Mbps, the corresponding performance is 0.2dB away from the performance in AWGN channel with BER equals to $10^{-4}$. When the transmission rates are increased to 25Mbps and 50Mbps, the gap between the performance of the proposed scheme and the performance in AWGN channel are 0.8dB and 1.2dB, respectively.

Fig. 8 illustrates the BER performance comparison between the proposed scheme, CIBS-CDMA [25] and MS-BS-PPM [16] in asynchronous UWB systems. The BER performance in the AWGN channel is given as a lower bound. The Walsh codes with length of 16 chips are used in CIBS-CDMA scheme. For the proposed scheme, the binary spreading sequence set with the ZCW of 3 applied for 8 users is given in Table II [30].

As the orthogonality of the spreading sequences in the proposed scheme can be maintained at the receiver, the perfor-
mance of the proposed BS-CDMA with the ZCW spreading sequence is more than 3dB better than the performance of the best known block spreading design, CIBS-CDMA, when the BER equals to $10^{-2}$. As shown in Fig. 8, the proposed scheme is about 1.2 dB away from the performance of an AWGN channel, this performance degradation is due to ISI within the data block, which can not be completely removed by the MMSE equalizer.

Fig. 9 makes the comparison between the BS-CDMA and conventional DS-CDMA scheme [19] in asynchronous UWB systems with various number of interferers. In the conventional DS-CDMA scheme, Kasami codes are used with length of 15. The MAI is approximated as Gaussian noise and suppressed statistically. However, this assumption is not realistic when the number of interferers is not large enough. It is shown that the proposed scheme performance is insensitive to the increasing number of users in contrast to the conventional DS-CDMA. For example, when BER equals to $10^{-2}$, the proposed scheme performs superior by 1.6dB, 2.2dB and 4.5dB to the conventional MMSE DS-CDMA scheme with 3, 5 and 8 coexisting users, respectively.

The analog spreading sequences with the ZCW of 3 are used for 3 and 5 users and they are shown in Fig. 10.

### C. Receiver Computation Complexity

For each user in the proposed scheme, there are total $2N_rN_cN_f$ multiplications in the MAI canceller and $N_c^2N_r^2+N_cN_r+2N_c+N_r^2$ multiplications in the FFT-based channel equalizer at the receiver side. Consequently, the complexity of the MMSE equalizers can be expressed as $O(N_c^2N_r^2)$, which indicates that the complexity of the proposed scheme is proportional to $N_r^2N_r^2$.

For the conventional BS-CDMA UWB receiver, a coefficient matrix of a linear MMSE equalizer, $\Gamma^\text{MMSE}_k$, is given in Eq. (30), where $PH_kH_k + \sigma^2I$ is an $(N_cN_r \times N_cN_r)$ matrix. According to linear algebra [19], the complexity of

---

**TABLE II**

**Binary Spreading Sequences with ZCW of 3 for 8 Users**

<table>
<thead>
<tr>
<th>User</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>UserI</td>
<td>+ + + + - - - - + + + +</td>
</tr>
<tr>
<td>UserII</td>
<td>+ - + - + + + + - - - +</td>
</tr>
<tr>
<td>UserIII</td>
<td>+ + + + - - - - + + + +</td>
</tr>
<tr>
<td>UserIV</td>
<td>+ + - - + + + + - - - +</td>
</tr>
<tr>
<td>UserV</td>
<td>+ - - - + + + + - - - +</td>
</tr>
<tr>
<td>UserVI</td>
<td>+ - + - + + + + - - - +</td>
</tr>
<tr>
<td>UserVII</td>
<td>+ - - - + + + + - - - +</td>
</tr>
<tr>
<td>UserVIII</td>
<td>- - - - + + + + - - - +</td>
</tr>
</tbody>
</table>

---

Fig. 8. The BER performance in the NLOS UWB channel impulse response model: there are 8 coexisting users; transmission rate is 50Mb/s; each data block consists of 100 symbols; the sampling rate is 5GHz.

Fig. 9. BER performances of block spreading CDMA and symbol-by-symbol detected DS-CDMA in asynchronous NLOS UWB systems. Three circumstances are investigated with 3, 5 and 8 coexisting users and the transmission rate is 50Mb/s per user.

Fig. 10. Analog spreading sequences with ZCW = 3.
an \((N_c N_s \times N_c N_r)\) matrix inversion is \(O((N_c N_s)^3)\), which is also the computation complexity of a conventional MMSE BS-CDMA UWB system.

Clearly, the complexity of the proposed receiver is much lower than the complexity of the conventional MMSE BS-CDMA UWB receiver, especially when the UWB channel matrix is very large. The low complexity feature of the proposed scheme is largely due to the fact that FFT-based block-circulant matrix decomposition in Eq. (26) converts a general matrix inversion into a diagonal matrix inversion as shown in Eqs. (30) and (31).

For example, when the transmission rate is 50Mbps, the data block size \(N_c\) is 100 and the minimum multipath resolvability \(T_c\) is 0.167ns, the computation complexity of the conventional BS-CDMA UWB receiver is about 6,000 times of the proposed scheme.

VI. CONCLUSION

In this paper, a novel block-spreading CDMA multiuser detection scheme for asynchronous UWB communication systems is considered. By applying the spreading sequences with zero correlation window, the orthogonality of spreading sequences are maintained in asynchronous UWB systems with frequent selective fading UWB propagation. This results in zero MAI at the receiver. In order to eliminate ISI with low computation complexity, the channel matrix is converted from a block-Toeplitz into a block-circulant form. Consequently, an FFT-based MMSE equalization scheme for UWB system is applied. It is important to note that the proposed scheme not only performs superior to the best known multiuser detection scheme for UWB systems but also significantly reduces the computation complexity compared to the conventional MMSE DS-CDMA and BS-CDMA UWB schemes.

APPENDIX (I): A SUMMARY OF ANALOG ZCW SEQUENCES GENERATION

Theoretically, [31] proves that a set of spreading codes with both zero auto-correlation sidelobes and zero cross-correlation does not exist. However, according to the theoretic limit, known as Welch bound, it is possible to design a set of sequences with a zero correlation window. Within this window, both zero auto-correlation sidelobes and zero cross-correlation can be simultaneously achieved.

It is assumed that the number of users is \(2N - 1\), where \(N\) is a positive integer. Vector \(s_k\) denotes the spreading sequence corresponding to the user \(k\) and is defined as

\[
s_k = \{s_{k1}, s_{k2}, \cdots, s_{kN_c}\}, k \in \{1, 2, \cdots, 2N - 1\}. \tag{I-1}
\]

According to the specified spreading-sequences-design-criteria in Eq. (19), we have \((2N - 1)! + (2N - 1)^2\) equations, which are summarized as

<table>
<thead>
<tr>
<th>(s_k^1 s_{k1} = 1)</th>
<th>(s_k^2 s_{k2} = 0)</th>
<th>(\cdots)</th>
<th>(s_k^{(2N-1)} s_{k(2N-1)} = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_k^2 s_{k1} = 1)</td>
<td>(s_k^3 s_{k2} = 0)</td>
<td>(\cdots)</td>
<td>(s_k^{(2N-1)} s_{k(2N-1)} = 0)</td>
</tr>
<tr>
<td>(s_k^3 s_{k1} = 0)</td>
<td>(s_k^4 s_{k2} = 0)</td>
<td>(\cdots)</td>
<td>(s_k^{(2N-1)} s_{k(2N-1)} = 1)</td>
</tr>
<tr>
<td>(s_k^4 s_{k1} = 0)</td>
<td>(s_k^5 s_{k2} = 1)</td>
<td>(\cdots)</td>
<td>(s_k^{(2N-1)} s_{k(2N-1)} = 0)</td>
</tr>
<tr>
<td>(\cdots)</td>
<td>(\cdots)</td>
<td>(\cdots)</td>
<td>(\cdots)</td>
</tr>
<tr>
<td>(s_k^{(2N-1)} s_{k1} = 1)</td>
<td>(s_k^{(2N-1)} s_{k2} = 0)</td>
<td>(\cdots)</td>
<td>(s_k^{(2N-1)} s_{k(2N-1)} = 0)</td>
</tr>
</tbody>
</table>

Theoretically, the minimum value of spreading gain \(N_f\) is denoted by \(N_{f,min}\) and can be obtained as

\[
N_{f,min} = \left[ \frac{(2N - 1)! + (2N - 1)^2}{2N - 1} \right] \leq 4N^2 - 2. \tag{I-2}
\]

Eq. (1-2) implies that there always exist \((2N - 1)\) sequences with ZCW of 3, when \(N_f\) is chosen to be \(4N - 2\).

Subsequently, the set of spreading sequences with ZCW of 3 is obtained by performing the computer search. As the entry of searched spreading sequences is not necessarily a digital binary code or integer, the obtained sequences is called analog spreading sequences.

APPENDIX (II): PROOF OF Eq. (38)

Proof: As \(H_k^T H_k\) is a circulant matrix given as

\[
H_k^T H_k = \begin{pmatrix}
\rho k_0 & \rho k_{(N_c - 1)} & \cdots & \rho k_1 \\
\rho k_1 & \rho k_0 & & \rho k_2 \\
\vdots & \vdots & \ddots & \vdots \\
\rho k_{(N_c - 1)} & \rho k_{(N_c - 2)} & \cdots & \rho k_0
\end{pmatrix} \tag{II-1}
\]

Its eigenvalue can be expressed as

\[
\lambda_{km} = \sum_{i=0}^{N_c - 1} \rho_{ki} e^{2\pi i mk/N_c}. \tag{II-2}
\]

Consequently, we get

\[
\sum_{m=1}^{N_c} \lambda_{km} = \sum_{i=0}^{N_c - 1} \rho_{ki} \sum_{m=1}^{N_c} e^{2\pi i mk/N_c}, \tag{II-3}
\]

As we have

\[
\sum_{m=1}^{N_c} e^{2\pi i mk/N_c} = \begin{cases} N_c & k = 0, \\
0 & \text{otherwise}. \tag{II-4}
\end{cases}
\]

the eigenvalues sum of matrix \(H_k^T H_k\) is given by

\[
\sum_{m=1}^{N_c} \lambda_{m} = G_H N_c. \tag{II-5}
\]

In order to maximize the SINR of the user \(k\) with constraint of Eq. (II-3), we introduce a Lagrange multiplier \(\mu_k\) and get

\[
\mathcal{L}(\mu_k, \lambda_{k1}, \cdots, \lambda_{kN_c}) = \sum_{j=1}^{N_c} \frac{P_{\lambda_{kj}}}{(P_{\lambda_{kj}} + \sigma^2)} \sum_{i=1}^{N_c} \frac{\sigma^2}{(P_{\lambda_{ki}} + \sigma^2)} + \mu_k (\sum_{i=1}^{N_c} \lambda_{ki} - H_c). \tag{II-6}
\]

The maximization of SINR is obtained when

\[
\frac{\partial (\mathcal{L})}{\partial (\lambda_{ki})} = (\sum_{j=1}^{N_c} \frac{\sigma^2}{(P_{\lambda_{kj}} + \sigma^2)})^2 - \mu_k = 0, i \in [1, N_c]. \tag{II-7}
\]
As a result, we have

\[
\frac{P}{(P\lambda_{k1} + \sigma_k^2)^2} = \frac{P}{(P\lambda_{k2} + \sigma_k^2)^2} = \cdots \frac{P}{(P\lambda_{kN_c} + \sigma_k^2)^2} = \cdots
\]

\[
= \frac{\mu_k}{\left( \sum_{j=1}^{N_c} \left( P\lambda_j + \sigma_j^2 \right) \right)^2}, \quad (II-8)
\]

or

\[
\lambda_{k1} = \lambda_{k2} = \cdots = \lambda_{kN_c} = G_H. \quad (II-9)
\]

REFERENCES


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He has been involved in the technical committee of several international conferences, such as ICC’05, PIMRC’05, WirelessCom’05 and so on.