Differential Modulation for Bidirectional Relaying With Analog Network Coding

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Abstract—In this correspondence, we propose an analog network coding scheme with differential modulation (ANC-DM) using amplify-and-forward protocol for bidirectional relay networks when neither the source nodes nor the relay knows the channel state information (CSI). The performance of the proposed ANC-DM scheme is analyzed and a simple asymptotic bit error rate (BER) expression is derived. The analytical results are verified through simulations. It is shown that the BER performance of the proposed differential scheme is about 3 dB away from that of the coherent detection scheme. To improve the system performance, the optimum power allocation between the sources and the relay is determined based on the simplified BER. Simulation results indicate that the proposed differential scheme with optimum power allocation yields 1–2 dB performance improvement over an equal power allocation scheme.

Index Terms—Amplify-and-forward protocol, analog network coding, bidirectional relaying, differential modulation.

I. INTRODUCTION

Bidirectional relay communication has attracted considerable interest recently [1]–[6], and various bidirectional relay protocols for wireless systems have been proposed [1]–[3]. In [1] and [2], the conventional network coding scheme is applied to the bidirectional relay network. Two source nodes transmit to the relay, separately. The relay decodes the received signals, performs binary network coding, and then broadcasts network coded symbols back to both source nodes. However, this scheme may cause irreducible error floor due to the detection errors which occur at the relay node.

In [3]–[5], an amplify and forward based network coding scheme, referred to as the analog network coding, was proposed. In this scheme, both source nodes transmit at the same time so that the relay receives a superimposed signal. The relay then amplifies the received signal and broadcasts it to both source nodes. Analog network coding is particularly useful in wireless networks as the wireless channel acts as a natural implementation of network coding by summing the wireless signals over the air.

Almost all existing works in bidirectional relay communications using analog network coding assume that the sources and the destination have perfect knowledge of channel state information (CSI) for all transmission links. As a result, coherent detection can be readily employed at either sources or relay, or both [1]–[5]. In some scenarios, e.g., the slow fading environment, the CSI is likely to be acquired by the use of pilot symbols. However, when the channel coefficients vary fast, channel estimation may become difficult. In addition, the channel estimation increases computational complexity in the relay node and reduces the data rate. Moreover, it would be difficult for the destination to acquire the source-to-relay channel perfectly through pilot signal forwarding without noise amplification. Hence, differential modulation without need of any CSI would be a practical solution.

In a differential bidirectional relay network, each source receives a superposition of differentially encoded signals from the other source, and it has no knowledge of CSI of both channels. All these problems present a great challenge for designing differential modulation schemes in two-way relay channels. In [6], differential receivers for differential two-way relaying were presented using analog network coding. These noncoherent schemes were realized by averaging the channel coefficients of the subtraction of adjacent received signals. However, the approaches may result in more than 3 dB performance loss compared to the coherent schemes due to instantaneous detection errors.

In this correspondence, we propose an analog network coding scheme with differential modulation (ANC-DM) using amplify-and-forward protocol for bidirectional relay networks so that the CSI is not required at both sources and the relay. The performance of the proposed ANC-DM scheme is analyzed and a simple asymptotic bit error rate (BER) expression is derived. The analytical results are verified through simulations. They show that the proposed differential scheme is about 3 dB away compared to the coherent detection scheme. To improve the system performance, the optimum power allocation between the sources and the relay is determined based on the provided simplified BER. Simulation results show that optimum power allocation yields 1–2 dB performance improvement over an equal power allocation scheme.

Note that unlike [6], the destination realizes differential detection by subtracting away its own contribution in the received signals based on the power estimation. Besides, our scheme requires linear complexity, while the detectors in [6] are much complex and nonlinear which may calculate the modified Bessel function of second kind. Note also that the application of differential modulation for the bidirectional relaying in [1] and [2] with digital network coding is relatively straightforward as the received signals at the relay from the sources can be decoded separately due to the use of orthogonal transmissions. However, for amplify-and-forward relaying [3]–[5], the relay receives a superposition of differentially encoded signals from the sources which makes the final detection at the source difficult.

The rest of the correspondence is organized as follows: In Section II, we describe the proposed differential modulation scheme. Theoretical analysis is given in Section III. Section IV presents the optimal transmit power allocation between the sources and the relay. Simulation results are provided in Section V. In Section VI, we draw the main conclusions.

Notation

Boldface lower-case letters denote vectors, \((\cdot)^T\) stands for complex conjugate, \((\cdot)^\dagger\) represents transpose, \(\text{E}\) is used for expectation, \(\text{Var}\) represents variance, \(|x|^2 = x^H x\), and \(\Re(\cdot)\) denotes real part.

II. DIFFERENTIAL MODULATION FOR BIDIRECTIONAL RELAY NETWORKS

A. Differential Encoding

We consider a three-node bidirectional relay network consisting of two source nodes, denoted by \(S_1\) and \(S_2\), and one relay node, denoted by \(R\). All nodes are equipped with one antenna and operate in a half-duplex way so that the complete transmission can be divided into two
phases, as shown in Fig. 1. In the first phase, both source nodes simultaneously send the differentially encoded information to the relay, and in the second phase, the relay broadcasts the combined signals to both sources. Let $c_1(t) \in \mathcal{A}$ denote the symbol to be transmitted by the source $S_1$ at the time $t$, where $\mathcal{A}$ represents a unity power $M$-PSK constellation set. In the differential modulation bidirectional relay system, the signal $s_1(t)$ sent by the source $S_1$ is given by

$$s_1(t) = s_1(t-1)c_1(t), c_1(t) \in \mathcal{A}. \quad (1)$$

Similarly, the signal transmitted by $S_2$ at the time $t$ is given by

$$s_2(t) = s_2(t-1)c_2(t), c_2(t) \in \mathcal{A}. \quad (2)$$

B. Differential Decoding

In the bidirectional relayed transmission, the source nodes first broadcast the information to the relay. For simplicity, we assume that the fading coefficients are constant over one frame of length $L$, and change independently from one frame to another. The received signal in the relay at time $t$ can be expressed as

$$y_r(t) = \sqrt{p_1}h_1s_1(t) + \sqrt{p_2}h_2s_2(t) + n_r(t) \quad (3)$$

where $p_1$ and $p_2$ represent the transmit power at $S_1$ and $S_2$ respectively, $h_1$ and $h_2$ are the Rayleigh fading coefficients with zero mean and unit variance between $S_1$ and $R$, and between $S_2$ and $R$, respectively, $n_r(t)$ denotes zero mean complex Gaussian random variable with two sided power spectral density of $N_0/2$ per dimension, and we furthermore assume $S_1$, $S_2$, and $R$ have the same noise variance.

In the second phase, the relay $R$ amplifies $y_r(t)$ by a factor $\beta$ and then broadcasts its conjugate, denoted by $y^*_r(t)$, to both $S_1$ and $S_2$. The corresponding signal received by $S_1$ at time $t$, denoted by $y_1(t)$, can be written as

$$y_1(t) = \beta \sqrt{p_1}h_1y^*_r(t) + n_1(t) = \mu s_1^*(t) + \nu s_2^*(t) + w_1(t) \quad (4)$$

where $\beta = (p_1|h_1|^2 + p_2|h_2|^2 + N_0)^{-1/2}$, $p_r$ represents the transmit power by the relay, $\mu \triangleq \beta \sqrt{p_1|h_1|^2}$, $\nu \triangleq \beta \sqrt{p_2|h_2|^2}$, and $w_1(t) \triangleq \beta \sqrt{p_rN_0}n_r^*(t) + n_1(t)$. Note that unlike traditional ANC schemes [3]–[5], $y^*_r(t)$ is transmitted from the relay, which obviously yields $\mu > 0$. The reason of doing this is to make the receiver easily estimate $\mu$ and then subtract $\mu s_1^*(t)$ in (4) for differential detection.

As the relay has no CSI, the normalization factor $\beta$ has to be obtained indirectly. We may rewrite the received signals in (3) in a vector format, given by

$$\mathbf{y}_r = \mathbf{y}_r, \mathbf{s}_1 = [s_1(1), \ldots, s_1(L)]^T, \mathbf{s}_2 = [s_2(1), \ldots, s_2(L)]^T, \text{and } \mathbf{n}_r = [n_r(1), \ldots, n_r(L)]^T. \quad (5)$$

To estimate the average receive power, we multiply the received signals by its Hermitian transpose as

$$||\mathbf{y}_r||^2 = p_1|h_1|^2 s_1^H s_1 + p_2|h_2|^2 s_2^H s_2 + 2\sqrt{p_1p_2} \Re\{h_1^*h_2s_1^H s_2\} + 2\sqrt{p_1p_2} \Re\{h_1 n_r^H s_1\} + 2\sqrt{p_2p_r} \Re\{h_2 n_r^H s_2\} + n_r^H n_r. \quad (6)$$

By taking the expectation of (6), $\beta$ can be then approximated at high SNR by

$$\beta = \sqrt{\frac{\mathbb{E}||\mathbf{y}_r||^2}{L}} \approx \sqrt{\frac{||\mathbf{y}_r||^2}{L}} \quad (7)$$

where $\mathbb{E}[\mathbf{s}_1^H \mathbf{s}_1] = \mathbb{E}[\mathbf{s}_2^H \mathbf{s}_2] = L$, $\mathbb{E}[\mathbf{n}_r^H \mathbf{n}_r] = LN_0$, and $\mathbb{E}[\mathbf{s}_1^H \mathbf{s}_2] = \mathbb{E}[\mathbf{s}_1^H \mathbf{n}_r] = \mathbb{E}[\mathbf{n}_r^H \mathbf{s}_2] = 0$.

Similarly, the received signal at $S_2$ can be calculated as

$$y_2(t) = \beta \sqrt{p_2}h_2y^*_r(t) + n_2(t). \quad (8)$$

As $S_1$ and $S_2$ are mathematically symmetrical, as shown in (4) and (8), for simplicity, we in the next only discuss the decoding as well as the corresponding theoretical analysis for signals received by $S_1$.

Recalling the differential encoding process in (2) and (4) can be further written as

$$y_1(t) = \mu s_1^*(t) + \nu s_2^*(t) + w_1(t), \quad (9)$$

where $y_1 = [y_1(1), \ldots, y_1(L)]^T$ and $w_1 = [w_1(1), \ldots, w_1(L)]^T$. At high SNR, we may approximately obtain

$$\mu^2 + ||\nu||^2 \approx \frac{||\mathbf{y}_1||^2}{L} \quad (10)$$

Since the source node $S_1$ can retrieve its own information $s_1(t-1)$ and $c_1(t)$, based on (1) and (9), we have the following transformation

$$\hat{y}_1(t) \triangleq c_1^*(t)y_1(t-1) - y_1(t) = \nu s_2^*(t-1) + c_1^*(t) - c_2(t) + \hat{\nu}_1(t) \quad (11)$$

where $\hat{\nu}_1(t) \triangleq c_1^*(t)c_1(t-1) + w_1(t)$. Then, $||\nu||^2$ can be approximately calculated in a similar way as (11)

$$||\nu||^2 \approx \frac{\mathbb{E}[||s_2(t-1)||^2]}{||\mathbf{c}_1(t) - c_2(t)||^2} \quad (12)$$

where $\mathbb{E}[||s_2(t-1)||^2] = 1$, and the calculation of $\mathbb{E}[||s_2(t-1)||^2]$ is given in Section II-C. As $\mu$ is positive, by combining (7), (11), and (13), we have

$$\mu \approx \left\{ \begin{array}{ll} \sqrt{\sum_{t} \Delta}, & \Delta > 0 \\ 0, & \Delta \leq 0 \end{array} \right. \quad (14)$$
where
\[ \Delta \triangleq \left( y_1^T y_1 \right) / \left( L - \left( \tilde{y}_1^T \tilde{y}_1 \right) / \left( \mathbb{E} \left[ \| s(t-1) \|^2 | c_1(t) - c_2(t) \|^2 \right] \right) \].
Note that in low SNR, \( \Delta \) could be negative due to the noise effect, and thus we set \( \mu \approx 0 \) instead. The estimation method given in (14) is evaluated in Fig. 2.

By subtracting \( \mu s_1(t) \), (9) can be further written as
\[ y_1(t) = \mu s_1(t) = \nu s_1(t - 1) c_2(t) + w_1(t) = (y_1(t - 1) - w_1(t - 1)) c_2(t) + w_1(t). \] (15)

Finally, the following linear decoder can be used to recover \( c_2(t) \)
\[ \tilde{s}_2(t) = \arg \max_{c_2(t) \in A} \Re \{ y_1(t) y_1^*(t - 1) c_2(t) \} \]. (16)

And \( c_1(t) \) can be differentially decoded in a similar way by the source \( S \). Note that in comparison to traditional differential modulation [7], the extra complexity comes from the \( \mu \) estimation in (14), which is linear and only comprises a few number of additions and multiplications. But the receiver in [6] requires complicated computations, such as the zeroth-order modified Bessel function of the second kind.

By ignoring the second-order term, the corresponding SNR of the proposed differential detection scheme can be written as
\[ \gamma_d \approx \frac{\| y_1 \|^2}{2 \text{var} \{ w_1(t) \}} \approx \frac{\beta^2 v p_1 | h_1 |^2 | h_2 |^2}{2 (\beta^2 p_1 N_0 | h_1 |^2 + N_0)} \]
\[ \approx \frac{\psi_s \psi_r | h_1 |^2 | h_2 |^2}{(\psi_s + \psi_r) | h_1 |^2 | h_2 |^2 + 1} \] (17)
where \( \text{var} \{ w_1(t) \} \) is the variance of symbols produced in the constellation by \( b_i(t) \) in (4), the corresponding SNR can be calculated as
\[ \gamma_c \triangleq \frac{\| y_1 \|^2}{\text{var} \{ w_1(t) \}} = \frac{\psi_s \psi_r | h_1 |^2 | h_2 |^2}{(\psi_s + \psi_r) | h_1 |^2 | h_2 |^2 + 1}. \] (18)

By comparing (17) and (18), we can easily obtain
\[ \gamma_d \approx \gamma_c \frac{\psi_r}{2} \] (19)
which clearly indicates that the differential detector in (16) suffers around 3 dB performance loss compared to the coherent scheme.

C. The Calculation of \( \mathbb{E}[|c_1(t) - c_2(t)|^2] \) in (13)

From (13), the average power of \( c_1(t) - c_2(t) \) needs to be calculated. When \( M \)-PSK constellations are applied, the number of symbols produced in the new constellation by \( c_1(t) - c_2(t) \) is finite. Hence, it is easy to derive the average power of the new constellation sets. Note that the value of \( c_1(t) - c_2(t) \) can be equal to zero, which may affect the estimation accuracy in (13).

In order to overcome this problem, we may properly choose a rotation angle for the symbol modulated in source \( S \) by \( c_2(t) e^{-j\theta} \), ensuring that \( c_1(t) - c_2(t) \) in (13) is nonzero. For a \( M \)-PSK constellation, the effective rotation angle is in the interval \( [-\pi/M, \pi/M] \) from the symmetry of symbols. For a regular and symmetrical constellation, the rotation angle may be simply set as \( \theta = \pi/M \). Similar approach may be used to generate the rotation angle for other types of constellations.

Here, we give two examples on how to compute the average symbol power:

1. Supposing BPSK constellation \( \{-1, 1\} \) is used, we have \( c_1(t) - c_2(t) \in \{-2, 0, 2\} \). Hence, the calculation of average power in the new set is straightforward.

2. Supposing \( S_1 \) uses the BPSK set \( \{-1, 1\} \), by constellation rotation, \( S_2 \) can use \( \{-j, j\} \). And we can get \( c_1(t) - c_2(t) \in \{-1 - j, -1 + j, 1 - j, 1 + j\} \). Hence, it is also easy to derive the average power in the new constellation set.

III. PERFORMANCE ANALYSIS

For simplicity, we in this section analyze the BER performance using BPSK for the proposed ANC-DM scheme, and we assume \( p_1 = p_2 = p_s \) and \( \psi_s = \lambda \psi_r \), where \( \lambda > 0 \), and thus, \( \psi_1 = \psi_2 = \psi_r = \lambda \psi_r \). Equation (17) can be rewritten as
\[ \gamma_d \approx \frac{\psi_s \psi_r | h_1 |^2 | h_2 |^2}{2 (1 + \lambda) (\psi_1 | h_1 |^2 + \psi_r | h_2 |^2 + 1)} \] (20)
where \( \psi_r \triangleq (1 + \lambda) \psi_r \).

Let \( X = \gamma_d \), and the BER for BPSK modulation can generally be expressed as
\[ \text{BER} = \mathbb{E}[Q(2X)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-x^2) F_X(x) \, dx \] (21)
where \( Q(\cdot) \) is the Gaussian-Q function, \( F_X(x) \) is the cumulative distribution function (CDF) of \( X \). The right side of the equation can be readily obtained by integration by parts. The above expression is useful as it allows us to obtain the BER directly in terms of the CDF of \( X \).

By using a general result from [8], the BER in (21) can be approximated in the high SNR regime by considering a first order expansion of the CDF of \( X \). Specifically, if the first order expansion of the CDF of \( X \) can be written in the form
\[ F_X(x) = \frac{\alpha x^{N+1}}{X^{N+1} + (X + 1)} + o(x^{N+1}) \] (22)
where \( X \) represents the average transmit SNR. At high SNR, the asymptotic BER is given by [8]
\[ \text{BER} \approx \frac{\alpha \Gamma \left( N + \frac{3}{2} \right)}{2 \sqrt{\pi X^{N+1}} + o \left( X^{-N+1} \right)} \] (23)
The PDF of \( X \) can be obtained with the help of [9]
\[ P_X(x) = \frac{(1 + \lambda)^2 \exp \left( -2(1 + \lambda)(\psi_s^{-1} + \psi_r^{-1}) x \right)}{\psi_s \psi_r} \times \frac{K_1 \left( \frac{4(1 + \lambda) x}{\sqrt{\psi_s \psi_r}} \right)}{U(2(1 + \lambda) x)} \] (24)
where \( K_0(\cdot) \) and \( K_1(\cdot) \) are the zeroth-order and first-order modified Bessel functions of the second kind, respectively, and \( U(\cdot) \) is the unit step function. Note that the exact BER, which is complicated in computation, does not have a closed-form solution. However, at high SNR, when \( \alpha \) approaches zeros, the \( K_1(\cdot) \) function converges to \( 1/\zeta \), and the value of the \( K_0(\cdot) \) function is comparatively small, which could be ignored for asymptotic analysis. Hence, \( P_X(x) \) in (24) can be approximated as
\[ P_X(x) \approx \frac{(1 + \lambda)(\psi_s^{-1} + \psi_r^{-1}) \exp \left( -2(1 + \lambda)(\psi_s^{-1} + \psi_r^{-1}) x \right)}{\psi_s \psi_r} \] (25)
For the ANC-DM, the CDF of destination SNR \( \gamma_{d} \) can be approximated as
\[ F_X(x) \approx 1 - \exp \left( -2(1 + \lambda)(\psi_s^{-1} + \psi_r^{-1}) x \right) \approx 2(1 + \lambda)(\psi_s^{-1} + \psi_r^{-1}) x + o(x^{1+\epsilon}). \] (26)
Finally, comparing (26) with (22) and (23), the asymptotic BER of ANC-DM at high SNR can be approximated as

$$\text{BER} \approx \frac{2(1+\lambda)}{2\sqrt{\pi}} \left(\psi^{-1}_\lambda + \psi^{-1}_\rho\right) = \frac{(1+\lambda)(\psi^{-1}_\lambda + \psi^{-1}_\rho)}{2}. \quad (27)$$

IV. TRANSMIT POWER ALLOCATION

In this section, we discuss how to allocate power to both sources and the relay subject to total transmission power constraint. It can be seen from (27) that the asymptotic BER of the proposed differential modulation scheme depends nonlinearly upon $p_\lambda$ and $p_\rho$. Hence, when the total transmit power is fixed, $2p_\lambda + p_\rho = p$, the power allocation problem over Rayleigh channels can be formulated to minimize the asymptotic BER at high SNR in (27)

$$\min \text{BER} \quad \text{s.t.} \quad 2p_\lambda + p_\rho = p \quad (0 < p_\lambda < p, 0 < p_\rho < p) \quad (28)$$

where we assume $p_\lambda = p_\rho = p$, and $p_\rho = \lambda p_\rho$.

The power allocation problem is to find $p_\rho$ such that the BER in (27) is minimized subject to the power constraint by solving the following optimization problem:

$$\mathcal{L}(p_\lambda) = \text{BER} + \xi \left(2p_\lambda + p_\rho - p\right) \quad (29)$$

where $\xi$ is a positive Lagrange multiplier. The necessary condition for the optimality is found by setting the derivatives of the Lagrangian in (29) with respect to $p_\lambda$ and $p_\rho$, equal to zero, respectively. Reusing the power constraint, we can calculate at high SNR that

$$p_\lambda = \frac{p}{4}, \quad p_\rho = \frac{p}{2} \quad (30)$$

which indicates the power allocated in the relay should be equal to the total transmit power at both sources in order to compensate the energy used to broadcast combined information in one time slot.

V. SIMULATION RESULTS

In this section, we provide simulation results for the proposed ANC-DM scheme. We also include corresponding coherent detection results for comparison. All simulations are performed for a BPSK modulation over the Rayleigh fading channels. The frame length is $L = 100$. For simplicity, we assume that $2p_\lambda + p_\rho = p = 3$, and $S_1$, $S_2$ and $R$ have the same noise variance $N_0$. The SNR $\psi_\rho$ can be then calculated as $\psi_\rho = p_\rho / N_0$.

Fig. 2 shows the simulated BER performance for differential and coherent ANC schemes in bidirectional relaying without using constellation rotation. Equal transmit power allocation is applied: $p_\lambda = p_\rho = p/3$. It can be seen that the differential scheme suffers about 3 dB performance loss compared to the coherent ANC scheme, which has been validated by (19). We also include the Genie-aided result by assuming that $p_\rho$ is perfectly known by the source such that traditional differential decoding can be performed. It shows from the results that there is still almost no performance loss using the estimation method in (14) which clearly justifies the robustness of the proposed differential decoder.

It is worthwhile mentioning that, in [6], it uses similar Genie-aided result as a benchmark as well. The major difference is that the detectors in [6] have much inferior performance than the Genie-aided result, and it has about 6 dB performance loss in comparison to the coherent detection scheme. However, our proposed detection algorithm has comparable performance with the genie-aided result, and only has 3 dB performance loss than the coherent detection results. This clearly indicates that our proposed method outperforms the differential detectors in [6]. The main performance loss in [6] is due to that incoherent detection approach is employed by statistically averaging off the impact of channel fading coefficients ignoring the instantaneous channel state information. But, our method utilizes differential detection relying on the operation with the previous received signals which could be more adaptive to variation of the channels.

In Fig. 3, we compare the analytical and simulated BER performance of the proposed differential modulation scheme. Equal transmit power allocation is also applied by setting $p_\lambda = p_\rho = p/3$. From the figure, it can be observed that at high SNR, the analytical BER derived by (27) converges to the simulated result, which justifies the validation of (27).

In Fig. 4, we examine the BER performance of the proposed differential modulation protocol with power allocation by setting $p_\lambda = p/2$ and $p_\rho = p/2$ subject to the total power constraint. From Fig. 4, it can be observed that with optimal power allocation, the proposed scheme obtains about 2 dB performance gain in comparison with the equal power allocation scheme at high SNR. We also compare the result by another power setting: $p_\lambda = 0.4p$ and $p_\rho = 0.2p$, and inferior result can be again observed compared to the optimal power allocation.

In Fig. 5, we plot the BER curves in terms of $\lambda = p_\rho/p_\lambda$ defined in (20) using different noise variance $N_0$, with the asymptotic BER constraint in (27). With the power constraint $2p_\lambda + p_\rho = p$, the SNR can be therefore calculated by $\psi_\rho = \lambda p / ((2\lambda + 1)N_0)$, where $p = 3$. It shows that best performance is obtained when $\lambda = 0.5$. In other words,
Fig. 4. Simulated BER performance by the proposed differential scheme using transmit power allocation.

Fig. 5. Simulated BER performance by the proposed differential scheme using transmit power allocation in term of $\frac{p_c}{21}, \frac{p_r}{61}, \frac{p_t}{112}$, with different noise variance $N_0$ and $\frac{p_s}{78}$. It can be observed that the asymptotic BER is very close to the simulated results for various SNR values, and they result in the same power allocation solution of $p_c = p/4$ and $p_r = p/2$.

In Fig. 6, we examine the BER results of the proposed differential modulation scheme without and with using constellation rotation, where the signal constellation used by $S_1$ is rotated by $\pi/2$ relative to that by $S_2$. It can be observed that the new result has very similar with the curve without rotating constellations. This indicates that using constellation rotation may not give system any gains given large frame length.

Fig. 7 shows the average normalized mean-square error (MSE) of $\hat{\mu}$ estimation in (14) as a function of the SNR, where the average normalized MSE is calculated as $\frac{1}{N} \sum_{k=1}^{N} (\hat{f}(k) - f(k))^2 / E[|f(k)|]$ and $\hat{f}(k)$ is the estimate of $f(k)$. It can be observed that the estimation is quite accurate, particularly at high SNR.

VI. CONCLUSION

In this correspondence, we have proposed a simple differential modulation scheme for bidirectional relay communications using analog network coding when neither sources nor the relay has access to channel state information. Simulation results indicate that there exist about 3 dB loss compared to the coherent detection scheme. Analytical BER is derived to validate the proposed method. In addition, based on the asymptotic BER at high SNR, an optimal power allocation between the sources and the relay was derived to enhance the system performance.

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Adaptive Relaying in Noncoherent Cooperative Networks
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Abstract—This correspondence considers a cooperative network in which binary frequency-shift-keying (BFSK) modulation is employed to facilitate noncoherent communications between a source and a destination with the help of relays. Proposed is an adaptive cooperative scheme that employs two thresholds as follows. One threshold is used to select retransmitting relays: a relay retransmits to the destination if its decision variable is larger than the threshold, otherwise it remains silent. The other threshold is used at the destination for detection: the destination marks a relay as a retransmitting relay if the decision variable corresponding to the threshold is used at the destination for detection. Otherwise, the relay remains silent. References [4] and [5] study asymptotic SNR thresholds to minimize the end-to-end (e2e) BER in coherent cooperative networks.

Cooperative (or relay) diversity has recently emerged as a promising technique to combat fading experienced in wireless transmission. In particular, the end-to-end (e2e) bit-error-rate (BER) performance in a wireless network can be improved by having nodes (users) in the network cooperate with each other [1]–[3]. Two of the most well-known cooperative protocols are amplify-and-forward (AF) and decode-and-forward (DF). With DF, relays decode the source’s messages, re-encode and re-transmit to the destination. However, with the DF protocol, cooperation does not achieve full diversity if the relays always re-transmit the received message. This is due to possible retransmission of erroneously decoded bits of the message by the relays. An approach to reduce retransmission of erroneous bits is based on the instantaneous signal-to-noise ratio (SNR) of the source-relay link. When the source-relay SNR is larger than a threshold, the probability of decoding error at the relay is negligible and hence the relay retransmits the message. Otherwise, the relay remains silent. References [4] and [5] study asymptotic SNR thresholds to minimize the e2e BER in coherent cooperative networks.

Most of the previous works assume that the receivers (at relays and destination) have perfect knowledge of channel state information (CSI) of all the transmission links propagated by their received signals. Such an assumption is unrealistic in fast fading environment. Moreover, the complexity of channel estimation increases with the number of relays. To overcome these disadvantages, noncoherent modulation and demodulation have been proposed and considered as more robust methods for both AF and DF protocols in relay processing (see, e.g., [6], for the discussion of possible applications of noncoherent modulation/demodulation in strongly resource-limited systems, such as sensor networks). References [7]–[10] focus on the differential phase-shift keying (DPSK) for both AF and DF protocols. Frequency shift keying (FSK) is another popular candidate in noncoherent communications. Reference [11] proposes a framework of noncoherent cooperative transmission for the DF protocol employing FSK signals. Due to the complexity of the nonlinear maximum-likelihood (ML) decoding, a suboptimal piecewise linear (PL) scheme was also proposed in [11]. However, the continuous retransmission of the relays in both ML and PL schemes can still induce error propagation, hence limiting the BER performance of the system [4].

This work is also concerned with noncoherent cooperative networks in which binary FSK (BFSK) is employed. The transmission protocol proposed in this correspondence is based on the use of two thresholds as follows. After receiving the signal in the first phase from the source, each relay decodes and retransmits if its decision variable is larger than the first threshold, $\theta_1^{10}$. Otherwise it remains silent in the second phase. At the destination, the decision variable corresponding to a given relay is larger than the second threshold, $\theta_2^{10}$, the destination marks the relay as a retransmitting relay. Otherwise it marks the relay as a silent relay. Finally, the destination combines all the signals from the retransmitting relays and from the source to make a final decision. In essence, the first threshold enables each relay to adapt its operation according to the instantaneous source-relay channel quality, while the second threshold helps the destination to decide on what would be the retransmitting relays in the second phase. The average BER of the proposed scheme is analytically derived for the case of a two-relay network. The optimal threshold values or jointly optimal threshold values and power allocation are determined to minimize the average BER. Numerical and simulation results verify that our obtained BER expression is accurate. Compared to the PL scheme in [11], our proposed scheme with the optimal thresholds or jointly optimal thresholds and power allocation provides a superior performance under different channel conditions. It should be mentioned that while our engineering framework is applicable for a general network with an arbitrary number of relays, deriving the average BER for more than two relays is very tedious and involved and hence it is not pursued in this correspondence.