Abstract—It has been shown that distributed turbo coding (DTC) can approach the capacity of a wireless relay network. In the existing DTC schemes, it is usually assumed that error free decoding is performed at a relay. We refer to this type of DTC schemes as perfect DTC. In this paper, we propose a novel DTC scheme. For the proposed scheme, instead of making a decision on the transmitted information symbols at the relay as in perfect DTC, we calculate and forward the corresponding soft information. We derive parity symbol soft estimates for the interleaved source information when only the a posteriori probabilities (APPs) of the information symbols are known. The results show that the proposed scheme can effectively mitigate error propagation due to erroneous decoding at the relay. Simulation results also confirm that the proposed scheme approaches the performance bound of a distributed 2-hop relay network at high SNR.

Index Terms—Cooperative diversity, distributed coding, multi-hop transmission, relay network, soft information relaying, turbo coding, wireless sensor network.

I. INTRODUCTION

In relay networks signals are transmitted from one terminal to another through a number of relays. The main advantage of doing so is a reduced signal transmit power [1]–[5]. Recently, this concept has been applied in cooperative wireless systems [6]–[7], where each mobile terminal receives signals from other mobiles and then relays them to the destination.

To improve the performance in a relay network, some cooperative and distributed coding schemes [8]–[13] have been developed recently. A distributed turbo coded (DTC) system is developed for a 2-hop relay network in [11]–[13]. The source broadcasts the coded signals to both the destination and relay. The relay then decodes the received signals, and interleaves them prior to the encoding. The signals received at the destination consist of coded information symbols transmitted from the source and coded interleaved information symbols transmitted from the relay. These two signals form a distributed turbo code [25]. It has been shown that such a coding strategy performs close to the theoretic outage probability bound of a relay channel [11]. However, in the existing DTC schemes, it is usually assumed that the relay can perform error-free decoding. We refer to such DTC schemes as perfect DTC. This scenario can be realized by automatic repeat request (ARQ) in the link from the source to the relay. However, the use of ARQ will reduce the system transmission throughput.

In this paper, we propose a distributed turbo coding scheme with soft information relaying (DTC-SIR). In the proposed scheme, instead of making decision on the transmitted information symbols at the relay as in [11]–[13], the relay calculates and forwards the soft estimates of the transmitted symbols. The relay first calculates the a posteriori probabilities (APPs) of the information symbols. Then it calculates the parity symbol soft estimates of the interleaved source information based on the APP of the information symbols. Analytical results show that the proposed scheme approaches the performance of the perfect DTC schemes at a high signal-to-noise ratio (SNR) and achieves a higher throughput than latter.

The rest of the paper is organized as follows. Section 2 introduces the system and channel models. Section 3 presents the proposed DTC-SIR scheme. Performance analysis and calculation of outage probability bounds are carried out in Sections 4 and 5, respectively. In Section 6, we present simulation results. Conclusions are drawn in Section 7.

II. SYSTEM MODEL

For simplicity, we consider a 2-hop relay system, consisting of one source, one relay and one destination. Fig.1 gives a block diagram of the 2-hop relay system with a direct link from the source to the destination.

A block diagram of a parallel concatenated distributed turbo coded system is shown in Fig.2. The transmitted source information binary stream, denoted by B, is represented by

$$B = (b_1, \ldots, b_k, \ldots, b_l)$$

where $b_k$ is the binary symbol transmitted at time $k$ and $l$ is the frame length.

The binary information sequence B is first encoded by a channel encoder. For simplicity, we consider a recursive
systematic convolutional code (RSCC) [25] with a code rate of 1/2.

Let \( C \) represent the corresponding codeword, given by

\[
C = (C_1, \ldots, C_k, \ldots, C_t)
\]

where \( C_k = (b_k, c_k) \) is the codeword of \( b_k, c_k \in \{0, 1\}, b_k \) is the information symbol and \( c_k \) is the corresponding parity symbol.

The binary symbol stream \( C \) is then mapped into a modulated signal stream \( S \). For simplicity, we consider a BPSK modulation. The modulated codeword, denoted by \( S \), is given by

\[
S = (S_1, \ldots, S_k, \ldots, S_t)
\]

where \( S_k = (s_k^I, s_k^Q), s_k^I, s_k^Q \in \{-1, 1\}, \) are the modulated information and parity signals transmitted by the source at time \( 2k - 1 \) and \( 2k \), respectively.

We assume that the source and relay transmit data through orthogonal channels [5]-[12]. For simplicity, we will concentrate on a time division multiplex [8]-[12], for which the source and relay transmit in separate time slots. The proposed scheme can be extended to other orthogonal transmission techniques in a straightforward manner.

The source first broadcasts the information to both the destination and relay. The received signals at the relay and destination, at time \( 2k - 1 \) and \( 2k \), denoted by \( y_{sr,k} \) and \( y_{sd,k} \), respectively, can be expressed as

\[
y_{sr,k} = \frac{\sqrt{P_{sr}}h_{sr}S_k + n_{sr,k}}{P_{sd}h_{sd}S_k + n_{sd,k}}
\]

\[
y_{sd,k} = \frac{\sqrt{P_{sr}}h_{sr}S_k + n_{sr,k}}{P_{sd}h_{sd}S_k + n_{sd,k}}
\]

where \( y_{sr,k} = (y_{sr,k}^I, y_{sr,k}^Q), y_{sd,k} = (y_{sd,k}^I, y_{sd,k}^Q), P_{sr} = P_s \cdot (G_{sr})^2, P_{sd} = P_s \cdot (G_{sd})^2 \) are the received signal power\(^1\) at the relay and destination, respectively, \( P_s \) is the source transmit power, and \( G_{sr} = (\frac{\lambda}{4\pi d_0^2}) \left( \frac{d_{sr}}{d_0} \right)^{-\kappa/2} \) \( G_{sd} = (\frac{\lambda}{4\pi d_0^2}) \left( \frac{d_{sd}}{d_0} \right)^{-\kappa/2} \) [21], [22] are the channel gains between the source and relay and that between the source and destination, respectively, \( d_{sr} \) and \( d_{sd} \) are the distances between the source and relay, and between the source and destination, respectively, \( d_0 \) is a reference distance, \( \lambda \) is the carrier wavelength and \( \kappa \) is a path loss factor with values typically in the range \( 2 \leq \kappa \leq 6 \). Also \( h_{sr} \) and \( h_{sd} \) are the fading coefficients between the source and relay and between the source and destination, respectively. They are modeled as zero-mean, independent circular symmetric complex Gaussian random variables. In this paper, we consider a quasi-static fading channel, for which the fading coefficients are constant within one frame and change independently from one frame to another. Furthermore, \( n_{sr,k} = (n_{sr,k}^I, n_{sr,k}^Q) \), \( n_{sd,k} = (n_{sd,k}^I, n_{sd,k}^Q) \) and they are zero mean complex Gaussian random variables with two sided power spectral density of \( N_0/2 \) per dimension. In this paper, we assume that all noise processes have the same variances, without loss of generality. Different noise variances can be taken into account by appropriately adjusting the channel gain.

In the standard DTC, the relay decodes the received signals from the source, interleaves the decoded information symbols, re-encodes and sends them to the destination. Let \( x_{r,k} \) represent the signal transmitted from the relay at time \( k \). It satisfies the following transmit power constraint,

\[
E(|x_{r,k}|^2) \leq P_2
\]

where \( P_2 \) is the transmitted power limit at the relay.

The corresponding received signal at the destination at time \( k \), denoted by \( y_{rd,k} \), can be written as

\[
y_{rd,k} = G_{rd}h_{rd}x_{r,k} + n_{rd,k}
\]

where \( G_{rd} \) is the channel gain between the relay and destination, \( h_{rd} \) is the Gaussian complex fading coefficient between the relay and destination and \( n_{rd,k} \) is a zero mean complex Gaussian random variable with two sided power spectral density of \( N_0/2 \) per dimension. The overall received signals at the destination consist of the coded source signal transmitted from the source given in (5) and the coded interleaved source signal transmitted from the relay given in (7). These two signals form a standard distributed turbo code.

**III. DISTRIBUTED TURBO CODING WITH SOFT INFORMATION RELAYING (DTC-SIR)**

In this section, we propose a DTC scheme with soft information relaying when imperfect decoding occurs at the relay. Rather than making a decision on the transmitted symbol at the relay as in the conventional schemes, we calculate the soft information of the received signals. The challenge is the derivation of the parity symbol soft estimates for the interleaved source information when only the APPs of the information symbols are known. We derive an expression for the parity symbol soft estimates based on the APPs of information symbols.

The block diagram of the DTC-SIR system is shown in Fig. 3. In the proposed scheme, the processing at the relay can be divided into two steps. The decoder first uses a maximum a posteriori probability (MAP) decoding algorithm to calculate the APPs of the information symbols. At the second step, the relay utilizes the derived APPs to calculate the parity symbol soft estimates for the interleaved information symbols.

**A. Calculation of the APPs for the information symbols**

Let \( y_{sr} = (y_{sr,1}, \ldots, y_{sr,l}) \) represent the received signal sequence at the relay, where \( y_{sr,k} \) are given in (4).

The decoder is based on a BCJR MAP decoding algorithm [16],[25]. The relay uses \( y_{sr} \) to calculate the APP of \( b_k, k =

\(^1\)Note that the gain is usually related to the power; however, for notational simplicity of subsequent exposure, we have opted to relate them to the amplitude, leading to the square of the gain.
\[ P(\tilde{c}_k = \tilde{w}_c | \mathbf{y}_{sr}, \mathbf{P}_B) = \sum_{m \in U(\tilde{c}_k = \tilde{w}_c)} P(\tilde{b}_k = \tilde{w}_b | \mathbf{y}_{sr}, \mathbf{P}_B, g(k-1) = m) P(g(k-1) = m | \mathbf{y}_{sr}, \mathbf{P}_B) \]
\[ = \sum_{m \in U(\tilde{c}_k = \tilde{w}_c)} P(\tilde{b}_k = \tilde{w}_b | \mathbf{y}_{sr}) P(g(k-1) = m | \mathbf{y}_{sr}, \mathbf{P}_B) \] (14)

\[ P(g(k) = m | \mathbf{y}_{sr}, \mathbf{P}_B) = \sum_{m'} P(g(k) = m | g(k-1) = m', \mathbf{y}_{sr}, \mathbf{P}_B) P(g(k-1) = m' | \mathbf{y}_{sr}, \mathbf{P}_B) \]
\[ = \sum_{m'} P(b(m, m') | \mathbf{y}_{sr}) P(g(k-1) = m' | \mathbf{y}_{sr}, \mathbf{P}_B) \] (15)

Let \( \mathbf{C} \) be the vector of parity symbols of \( \tilde{\mathbf{B}} \), denoted by
\[ \tilde{\mathbf{C}} = (\tilde{c}_1, \ldots, \tilde{c}_k, \ldots, \tilde{c}_l) \] (10)
where \( \tilde{c}_k \) is the parity symbol of \( \tilde{b}_k \).

Let
\[ \mathbf{P}_B = \{ P(\tilde{b}_k = w | \mathbf{y}_{sr}), w = 0, 1, k = 1, \ldots, l \} \] (11)
represent the set of the APPs of the information symbols.

Let
\[ \tilde{\mathbf{P}}_B = \{ P(\tilde{b}_k = w | \mathbf{y}_{sr}), w = 0, 1, k = 1, \ldots, l \} \] (12)
where \( P(\tilde{b}_k = w | \mathbf{y}_{sr}) = P(b_{pk} = w | \mathbf{y}_{sr}) \) is the APP of the \( k \)-th interleaved information symbol, which is also the APP of the \( p_k \)-th un-interleaved information symbol.

Now let us calculate the APP of \( \tilde{c}_k \) given \( \mathbf{P}_B \), or equivalently, given \( \tilde{\mathbf{P}}_B \), denoted by
\[ P(\tilde{c}_k = \tilde{w}_c | \mathbf{y}_{sr}, \mathbf{P}_B, \tilde{\mathbf{P}}_B), w_c \in \{0, 1\} \] (13)

We calculate this probability in the following recursive format as shown in (14) and (15) at the top of the page, where \( U(\tilde{c}_k = \tilde{w}_c) \) is the set of branches, for which the output parity symbol is equal to \( \tilde{w}_c \), \( \tilde{w}_b \) is the output information symbol corresponding to the parity symbol \( \tilde{w}_c \) and the trellis state \( g(k-1) = m \). \( P(\tilde{b}_k = \tilde{w}_b | \mathbf{y}_{sr}, \mathbf{P}_B, g(k-1) = m) \) represents the APP of information symbol \( \tilde{w}_b \) at time \( k \), which is equivalent to \( P(\tilde{b}_k = \tilde{w}_b | \mathbf{y}_{sr}) \). \( P(g(k-1) = m | \mathbf{y}_{sr}, \mathbf{P}_B) \) is the probability of the state \( m \) at time \( k-1 \), \( b(m, m') \) represents the input information symbol resulting in the transition from state \( m \) at time \( (k-1) \) to state \( m' \) at time \( k \) and \( P(b(m, m') | \mathbf{y}_{sr}) \) is the APP of information symbol \( b(m, m') \), at time \( k \).

We assume that the encoder clears the register at the end of the encoding operation for each codeword, resulting in the initial state probabilities
\[ P(g(0) = 0) = 1 \text{ and } P(g(0) = m, m \neq 0) = 0 \] (16)

From (14) and (15), we can recursively calculate the APPs of \( \tilde{\mathbf{C}} \) by using the APPs of \( \mathbf{B} \). In (14)-(16), we only consider a forward recursive calculation of the APP of \( \tilde{\mathbf{C}} \). A more exact calculation method can be derived by using both forward and backward recursive operations, in which more calculations are needed.

We assume that in the considered BPSK modulation signal set, the binary bit 0 and 1 are mapped into 1 and \(-1\), respectively.
respectively. The parity symbol soft estimates of \( \tilde{c}_k \), \( k = 1, \ldots, l \), denoted by \( \tilde{s}_k^p \), can then be calculated as follows,

\[
\tilde{s}_k^p = P(\tilde{c}_k = 0|y_{sr}, P_B) \cdot 1 + P(\tilde{c}_k = 1|y_{sr}, P_B) \cdot (-1) \quad (17)
\]

We can write \( \tilde{s}_k^p \) as \( \tilde{s}_k^p = \tilde{s}_k^p + \tilde{n}_k \), where \( \tilde{s}_k^p \) is the exact parity symbol of \( b_k \) and \( \tilde{n}_k = \tilde{s}_k^p - \tilde{s}_k^p \) is an equivalent noise, which can be modeled as a Gaussian random variable with a zero mean and variance \( \sigma_n^2 \).

If \( \tilde{s}_k^p \) and \( \tilde{n}_k \) are independent, then the average power of \( \tilde{s}_k^p \) is

\[
E(|\tilde{s}_k^p|^2) = E(|\tilde{s}_k^p|^2 + |\tilde{n}_k|^2) = 1 + \sigma_n^2 > 1 \quad (18)
\]

However, we can note from (17) that \(-1 \leq \tilde{s}_k^p \leq 1\) and \(|\tilde{s}_k^p|^2 \leq 1\), which contradicts (18). This means that \( \tilde{s}_k^p \) and \( \tilde{n}_k \) are not independent. We introduce another equivalent model of \( \tilde{s}_k^p \) and write \( \tilde{s}_k^p \) in the following manner

\[
\tilde{s}_k^p = \tilde{s}_k^p (1 - \tilde{n}_k) \quad (19)
\]

where \( \tilde{n}_k \geq 0 \) is an equivalent noise, with the mean

\[
\mu_n = \frac{1}{l} \sum_{k=1}^{l} \tilde{n}_k = \frac{1}{l} \sum_{k=1}^{l} (1 - \tilde{s}_k^p \tilde{s}_k^p) = \frac{1}{l} \sum_{k=1}^{l} |\tilde{s}_k^p - \tilde{s}_k^p| \quad (20)
\]

and variance

\[
\sigma_n^2 = \frac{1}{l} \sum_{k=1}^{l} (1 - \tilde{s}_k^p \tilde{s}_k^p - \mu_n)^2 \quad (21)
\]

The signals transmitted from the relay can then be written as

\[
x_{r,k} = \beta \tilde{s}_k^p = \beta \tilde{s}_k^p (1 - \tilde{n}_k) \quad (22)
\]

where \( \beta \) is a normalization factor calculated from the transmitted power constraint at the relay

\[
E(|x_{r,k}|^2) = \beta^2 (1 - 2\mu_n + E(\tilde{n}_k^2)) = \beta^2 ((1 - \mu_n)^2 + \sigma_n^2) \leq P_2 \quad (23)
\]

where \( P_2 \) is the transmitted power limit at the relay.

Let \( \hat{\omega} = (1 - \mu_n)^2 + \sigma_n^2 \), then from (23), we can obtain

\[
\beta \leq \sqrt{\frac{P_2}{\hat{\omega}}} \quad (24)
\]

Since the source and the relay are transmitted in different time slots, the destination receiver can separate them. The destination received signal, corresponding to the relay signal at time \( k \), denoted by \( y_{rd,k} \), can be written as

\[
y_{rd,k} = G_{rd} h_{rd} x_{r,k} + n_{rd,k} = G_{rd} h_{rd} \beta \tilde{s}_k^p (1 - \mu_n) + \tilde{n}_{rd,k} \quad (25)
\]

where \( \tilde{n}_{rd,k} = n_{rd,k} - G_{rd} h_{rd} \beta \tilde{s}_k^p \tilde{n}_k - \mu_n \) is the equivalent noise at the destination for DTC-SIR, with a zero mean and variance of \( \sigma_{\tilde{n}}^2 \), given by

\[
\sigma_{\tilde{n}}^2 = N_0 + G_{rd}^2 |h_{rd}|^2 \beta^2 \sigma_n^2 \quad (26)
\]

The destination received signal, corresponding to the signal transmitted from the source is given in (5). From (5) and (25), we can observe that an overall distributed codeword consists of the coded information symbols transmitted from the source, given in (5), and the parity symbols of the interleaved information sequence sent from the relay, shown in (25). These two signals at the destination are denoted by \( y_{sd,k} \) and \( y_{rd,k} \), respectively. They are decoded by two separate decoders.

A turbo iterative decoding algorithm is performed between these two decoders. Fig. 4 shows a block diagram of the iterative decoder. Iterations are performed between the two decoders associated with \( y_{sd,k} \) and \( y_{rd,k} \). The MAP decoders with input symbols \( y_{sd,k} \) and \( y_{rd,k} \) calculate the a posteriori probability (APP) and extrinsic information of the transmitted information symbols and the interleaved information symbols, respectively. The extrinsic information of one decoder is used to update the a priori probability of the other decoder in the next iteration. After several iterations, the decision is made based on the APPs of the first decoder.

We should point out that the calculation of variable \( k(m, m') \) in the decoder, associated with \( y_{rd,k} \), is somewhat different from the (8) and is given by

\[
\gamma_k(m, m') = \exp \left( -\frac{|y_{rd,k} - G_{rd} h_{rd} \beta \tilde{s}_k^p (1 - \mu_n)|^2}{\sigma_{\tilde{n}}^2} \right)
\]

IV. PERFORMANCE ANALYSIS OF THE DTC-SIR

In this section, we analyze the performance of DTC-SIR. The calculation of the traditional union bound requires knowing the number of codewords with various Hamming weights, which needs an exhaustive search of the code trellis. Due to a high complexity of this search, we consider an average upper bound [20].

Let us first calculate the average destination received SNR for DTC-SIR and perfect DTC, respectively.

Let \( \gamma_{r'in} \) and \( \gamma_{r'out} \) be the input and output signal-to-noise ratio (SNR) at the relay decoder, respectively, where the input and output signals at the relay decoder are the received signals at the relay and the parity symbol soft estimate \( \tilde{s}_k^p \), given in (17), respectively. The relationship between \( \gamma_{r'in} \) and \( \gamma_{r'out} \) can be represented by a function \( f(x) \)

\[
\gamma_{r'out} = f(\gamma_{r'in}) \quad (27)
\]

where \( \gamma_{r'in} = \frac{P_{r'in} h_{sr}^2}{N_0} \) and \( \gamma_{r'out} = \frac{(1-\mu_n)^2}{\sigma_n^2} \), which can be calculated from (19)-(21).

For a convolutional code, the asymptotic soft decision coding gain at high SNR can be approximated by

\[
\gamma_c = 10 \log(R d_{free}) dB \quad (28)
\]

where \( R \) is the code rate and \( d_{free} \) is the code minimum free distance. This gain goes down with reducing the SNR. Therefore, for convolutional codes, the decoder SNR input/output

---

**Fig. 4.** Block diagram of Decoder
\[ p(H_2^{\lambda_p}) = \frac{2H_2^{\lambda_p}\exp \left(-\frac{H_2^{\lambda_p}}{\sqrt{Rd_{free} \gamma_{s-r}} d} + \frac{1}{\gamma_{r-d}} \right)}{Rd_{free} \sqrt{\gamma_{s-r} d}} \cdot K_1 \left( \frac{2H_2^{\lambda_p}}{\sqrt{Rd_{free} \gamma_{s-r}} d} \right) + 2K_0 \left( \frac{2H_2^{\lambda_p}}{\sqrt{Rd_{free} \gamma_{s-r}} d} \right) \]  

(31)

\[ \tilde{\gamma}_{RD} = \frac{1}{2} \int_0^\infty H_2^{\lambda_p} p(H_2^{\lambda_p}) dH_2^{\lambda_p} = \frac{32}{15} \left( \frac{1}{\sqrt{Rd_{free} \gamma_{s-r}}} + \frac{1}{\sqrt{\gamma_{r-d}}} \right)^{6} F \left( 3, \frac{3}{2}; \frac{3}{2}; \left( \frac{\sqrt{Rd_{free} \gamma_{s-r}} - \sqrt{\gamma_{r-d}}} {\sqrt{Rd_{free} \gamma_{s-r}} + \sqrt{\gamma_{r-d}}} \right)^2 \right) \]

+ \frac{32 Rd_{free} \gamma_{s-r} + \gamma_{r-d}}{5} \left( \frac{1}{\sqrt{Rd_{free} \gamma_{s-r}}} + \frac{1}{\sqrt{\gamma_{r-d}}} \right)^{8} F \left( 4, \frac{3}{2}; \frac{3}{2}; \left( \frac{\sqrt{Rd_{free} \gamma_{s-r}} - \sqrt{\gamma_{r-d}}} {\sqrt{Rd_{free} \gamma_{s-r}} + \sqrt{\gamma_{r-d}}} \right)^2 \right) \]  

(32)

The average \( \tilde{\gamma}_{RD} \) can be calculated from (32) at the top of the page [26], where \( F(a, b; c; d) \) is a hypergeometric function.

For perfect DTC, the SNR at the destination, corresponding to the signal transmitted from the relay, denoted by \( \gamma_{RD} \), is given by

\[ \gamma_{RD}^{\text{Perfect}} = \frac{P_{rd}|h_{rd}|^2}{N_0} = \gamma_{r-d} |h_{rd}|^2 \]  

(33)

Its average value, denoted by \( \tilde{\gamma}_{RD}^{\text{Perfect}} \), can be calculated as

\[ \tilde{\gamma}_{RD}^{\text{Perfect}} = E \left( \frac{P_{rd}|h_{rd}|^2}{N_0} \right) = \gamma_{r-d} \]  

(34)

Now let us calculate the average error probability upper bound of the DTC-SIR.

As stated in Section 3, a codeword of the distributed turbo coded system, consists of the coded information symbols \( S \), transmitted from the source and the parity of the interleaved information symbols, denoted by \( S_r \), sent from the relay. These two signals have different SNRs at the destination, denoted by \( \gamma_{SD} \) and \( \gamma_{RD} \), respectively.

Assume that an all-zero codeword is transmitted, then the pairwise error probability (PEP) that the decoder decides in favor of another erroneous codeword with Hamming weight \( d \), for linear codes, is given by [13], [19]

\[ P(d|h_{sd}, h_{sr}, h_{rd}) = Q(\sqrt{2d_1 \gamma_{SD} + 2d_2 \gamma_{RD}}) \]  

(35)

where \( \gamma_{RD} \) is given by (30), \( \gamma_{SD} = \frac{P_{sd}|h_{sd}|^2}{N_0} = \gamma_{s-d} |h_{sd}|^2 \), \( \gamma_{s-d} = \frac{P_{sd}}{N_0} \), \( d_1 \) and \( d_2 \) are the Hamming weights of the erroneous codewords with Hamming weight \( d \), transmitted from the source and the relay, respectively, such that \( d = d_1 + d_2 \).

The average sequence error probability can be derived by averaging (35) over the fading coefficients. Let \( P(d) \) be the average probability of decoding an erroneous code sequence with weight \( d \), then

\[ P(d) = \int_{h_{sd}} \int_{h_{sr}} \int_{h_{rd}} P(d|h_{sd}, h_{sr}, h_{rd}) p(h_{sd}) p(h_{sr}) p(h_{rd}) \]

\[ d(h_{sd}) d(h_{sr}) d(h_{rd}) \leq \frac{1}{2} (d_1 \gamma_{s-d})^{-1} f(d_2) \]  

(36)

where \( p(h_{sd}), p(h_{sr}), \) and \( p(h_{rd}) \) are the pdf of \( h_{sd}, h_{sr} \) and \( h_{rd} \).
\[ f(d_2) = \int_{h_{rd}} h_{sr} Q(\sqrt{2d_2\gamma_{RD}}) p(h_{rd})p(h_{sr})d(h_{rd})d(h_{sr}) \]
\[ = \int Q(\sqrt{4d_2H_2^p}) p(H_2^p) dH_2^p \]

where \( p(H_2^p) \) is given in (31).

The closed form expression of \( f(d_2) \) can be calculated by using the moment generating function (MGF) of the Harmonic mean of two exponential random variables [5], [26]. The exact closed form expression of \( f(d_2) \) is too complex to be presented here. At high SNR, \( f(d_2) \) can be approximated as [14], [18]

\[ f(d_2) \approx \left( \frac{1}{R_d\gamma_{SR}} + \frac{1}{\gamma_{rd}} \right) (d_2)^{-1} \]  

(38)

Let \( P_b \) be the average upper bound on the bit error rate (BER). At high SNR, \( P_b \) can be approximated as

\[ P_b \leq \sum_{d=d_{min}}^{3d} \sum_{i=1}^{l} p(d|i)P(d) \]  

(39)

\[ \approx \frac{1}{2} (\gamma_{rd})^{-1} \left( \frac{1}{R_d\gamma_{SR}} + \frac{1}{\gamma_{rd}} \right) \sum_{d=d_{min}}^{3d} \sum_{i=1}^{l} p(d|i)(d_1d_2)^{-1} \]

where \( l \) is the number of words with Hamming weight \( i \) and \( p(d|i) \) is the probability that an input word with Hamming weight \( i \) produces a codeword with Hamming weight \( d \).

Similarly, for perfect DTC, the average PEP of incorrectly decoding to a codeword with weight \( d \), denoted by \( p_{\text{perfect}}(d) \), can be calculated as

\[ p_{\text{perfect}}(d) \leq \frac{1}{2} (d_1\gamma_{rd})^{-1}(d_2\gamma_{rd})^{-1} \]  

(40)

Let \( P_b^{\text{Perfect}} \) be the average upper bound on the bit error rate (BER) of perfect DTC. It can be expressed as

\[ P_b^{\text{Perfect}} \leq \sum_{d=d_{min}}^{3d} \sum_{i=1}^{l} p(d|i)P^{\text{Perfect}}(d) \]  

(41)

\[ \approx \frac{1}{2} (\gamma_{rd})^{-1} \sum_{d=d_{min}}^{3d} \sum_{i=1}^{l} p(d|i)(d_1d_2)^{-1} \]

Due to imperfect decoding at the relay, DTC-SIR has a performance loss compared to perfect DTC. We define \( SNR_{\text{loss}} \) as the difference in SNR to achieve the same BER between DTC-SIR and perfect DTC. Comparing (39) and (41), we can see that the SNR loss of the DTC-SIR compared to the perfect DTC, denoted by \( SNR_{\text{loss}} \), can be approximated as

\[ SNR_{\text{loss}} \approx \frac{\gamma_{rd}}{R_d\gamma_{SR}} + 1 = \frac{1}{R_d\gamma_{Gap}} + 1 \]  

(42)

where \( \gamma_{Gap} = \gamma_{sr}/\gamma_{rd} \) and

\[ SNR_{\text{Gap}}(dB) = 10\log(\gamma_{Gap}) = 10\log(\gamma_{sr}/\gamma_{rd}) \]  

(43)

represent the difference in SNR between \( \gamma_{sr} \) and \( \gamma_{rd} \) in decibels.

From (42), we can observe that in order to decrease the gap between DTC-SIR and perfect DTC, we should make \( Rd_{\text{free}} \) as large as possible and the SNRGap between \( \gamma_{sr} \) and \( \gamma_{rd} \) as large as possible.

We should point out that the union bound is tight for independent fast fading channels only [17]-[19]. In order to get a tight upper bound over quasi-static fading channels, we should limit the conditional upper bound in (36), (39) and (41) before averaging over the fading coefficients [19], but no closed-form expression can be obtained, as shown in [19]. Since we are only interested in the relative performance loss of DTC-SIR compared to perfect DTC, shown in (42), the average union bound is sufficient for evaluating this performance loss. The calculation of the exact upper bound using the “limit before averaging” method from [19] is out of the scope of this paper.

V. OUTAGE PROBABILITY BOUND OF SOFT INFORMATION RELAYING

In this section, we calculate the outage probability bound of a soft information relaying (SIR). It is obvious that the outage probability is lower bounded by the perfect relay (PR) 2 and upper bounded by the amplify and forward relaying protocol (AAF) 3.

Let \( P_{\text{out}}^{\text{SIR}} \), \( P_{\text{out}}^{\text{PR}} \) and \( P_{\text{out}}^{\text{AAF}} \) denote the outage probability of SIR, PR and AAF, respectively, then we have

\[ P_{\text{out}}^{\text{PR}} < P_{\text{out}}^{\text{SIR}} < P_{\text{out}}^{\text{AAF}} \]  

(44)

For the AAF, the maximum average mutual information in bit/s/Herz, denoted by \( I_{AF} \), is given by [24]

\[ I_{AF} = \frac{T_s}{T_s + T_r} \log \left( 1 + \gamma_{sr}h_{sr}^2 + \gamma_{sr-}\gamma_{rd}h_{rd}^2 \right) \]  

(45)

where the coefficient \( \frac{T_s}{T_s + T_r} \) is due to the assumption of a time division duplex format, \( T_s \) and \( T_r \) represent the transmission duration from the source and the relay, and

\[ \gamma_{AF} = \frac{\gamma_{sr} - \gamma_{rd}}{\gamma_{sr}h_{sr}^2 + \gamma_{rd}h_{rd}^2 + 1} \]  

(46)

For simplicity, we assume that

\[ \gamma_{sr} = \gamma_{rd} = \gamma \text{ and } \gamma_{sr} = \gamma_{Gap} \gamma \]  

(47)

where \( \gamma_{Gap} = \gamma_{sr}/\gamma_{rd} \). Substituting (47) into (45), we get at high SNR,

\[ I_{AF} = \frac{T_s}{T_s + T_r} \log \left( 1 + \gamma h_{rd}^2 + \frac{\gamma_{Gap}^2h_{sr}^2h_{rd}^2}{\gamma_{Gap}\gamma h_{sr}^2 + \gamma h_{rd}^2 + 1} \right) \]  

(48)

From (48), the outage event for a spectral efficiency \( \eta \) is determined by \( I_{AF} < \eta \). The outage probability for Rayleigh fading channels, at high SNR, can be approximated as [24]

\[ P_{\text{out}}^{\text{AF}}(\gamma, \eta) := \text{P}(I_{AF} < \eta)^{\gamma \rightarrow \infty} \left( \frac{\gamma_{Gap}+1}{\gamma_{Gap}} \right)^{\frac{\eta(1+T_s/T_r)}{2\gamma_{Gap}} - 1} \]  

(49)

2PR is a relay scheme for which relay can always perform an error-free decoding

3AAF is a relay scheme, for which the relay amplifies the received signals and forwards it to the destination.
Fig. 5. BER performance comparison at $\gamma_{s-r}=10$dB

For the perfect relay, $\gamma_{Gap} \to \infty$, and (49) is hence approximated to

$$P_{PR}^{out}(\gamma, \eta) := \frac{1}{2} \left( \frac{2^{\eta(1+T_r/T_s)} - 1}{\gamma} \right)^2$$  \hspace{1cm} (50)$$

From (49) and (50), we can get the lower and upper bound of the SIR as follows,

$$P_{SIR}^{out}(\gamma, \eta) < P_{AAF}^{out}(\gamma, \eta) = \left( \frac{1 + \gamma_{Gap}}{2\gamma_{Gap}} \right) \left( \frac{2^{\eta(1+T_r/T_s)} - 1}{\gamma} \right)^2$$  \hspace{1cm} (51)$$

$$P_{SIR}^{out}(\gamma, \eta) > P_{PR}^{out}(\gamma, \eta) = \frac{1}{2} \left( \frac{2^{\eta(1+T_r/T_s)} - 1}{\gamma} \right)^2$$  \hspace{1cm} (52)$$

It is clear from the above expressions that the upper bound can approach the lower bound as $\gamma_{Gap} \gg 1$.

VI. SIMULATION RESULTS AND DISCUSSIONS

In this section, we provide simulation results comparisons for various DTC schemes. All simulations are performed for the BPSK modulation and a frame size of 130 symbols over quasi-static fading channels. We use a 4-state recursive systematic convolutional code (RSC) with the code rate of $1/2$ as the turbo component code. The generator matrix of the RSC is $(1, 5/7)$ with $d_{free}$ of 5. Based on (42), we can calculate the SNR loss of DTC-SIR, compared to the perfect DTC as

$$SNR_{loss} = \frac{2\gamma_{r-d}}{5\gamma_{s-r}} + 1 = \frac{2}{5\gamma_{Gap}} + 1$$  \hspace{1cm} (53)$$

For simplicity, we also assume that $P_{rd}$ and $P_{sd}$ are the same. Therefore $\gamma_{s-d}$ is equal to $\gamma_{r-d}$. We carry out the performance comparisons under the following two scenarios.

A. Comparison for various $\gamma_{s-r}$

In this case, we investigate the performance of the DTC schemes when signal power from the source to the relay is fixed, while the power from the source to the destination and from the relay to the destination are varied.

For the DTC with no ARQ, the relay re-encodes the decoded codeword and passes it to the destination regardless weather the relay can make a correct decoding or not. Therefore, when decoding errors occur at the relay, the relay will make an erroneous symbol decision, and re-encoding the erroneous decoded symbols will cause error propagation. We refer to such scheme as the DTC with no ARQ. In the simulations, we use a Log-Map decoding algorithms at the destination.
Unlike the DTC-SIR, in which the equivalent noise variance, corresponding to the destination noise variance and the error variance in soft estimates, can be calculated as shown in (26), for DTC with no ARQ, the destination decoder has no knowledge of erroneous codewords introduced by the relay. Thus the decoder only gets the destination noise variance from the destination in the decoding. This will further degrade its performance.

As stated in Section 1, the performance of DTC can be improved by using an automatic retransmission request (ARQ) scheme in the link from the source to the relay. Such a scheme performs very close to perfect DTC. We refer to this scheme as DTC with ARQ. In the following simulation results, for DTC with ARQ, we assume that the maximum number of retransmissions (MNR) is 3 and a transmission is said to be a failure if it is still unsuccessful after the maximum number of retransmissions. In this case the relay will discard the frame.

Figs. 5 -7 compare the BER performance of DTCs with and without ARQ, and the proposed DTC-SIR, as a function of $\gamma_{r-d}$ and with $\gamma_{s-r}$ as a parameter. It can be noted from these figures that due to error propagation, DTC with no ARQ performs badly and achieves an error floor for high $\gamma_{r-d}$, especially at lower $\gamma_{s-r}$. In contrast, DTC-SIR can effectively mitigate the effect of error propagation. Its performance approaches DTC with ARQ as $\gamma_{s-r}$ increases.

Figs. 5 -7 also compare the performance of the DTC-SIR with the equivalent BER of outage probability lower and upper bounds, described by the perfect relay and the AAF relay system, as discussed in Section V. Here, the equivalent BER upper bounds are obtained by keeping the same gaps between the bounds and perfect DAF relays in FER and BER curves. It can be noted from the figures that DTC-SIR can approach the upper bound for larger $\gamma_{r-d}$ within 2dB for any values of $\gamma_{s-r}$ and the lower bound within 2dB at the BER of $10^{-4}$, at $\gamma_{s-r} = 25dB$.

From (42) and (53), we can also observe that if we keep $\gamma_{s-r}$ constant and vary $\gamma_{r-d}$, then the SNR loss of DTC-SIR compared to perfect DTC will increase as $\gamma_{r-d}$ increases. Fig. 8 compares the analytical SNR loss derived from (53) and the one obtained from simulation results in Figs. 5-7. It can be observed from Fig. 8 that the expression in (53) is tight for large $\gamma_{s-r}$. This is because the approximation of the soft decision coding gain $\gamma_c$ in (28) is tight only for high $\gamma_{r,in}$ or $\gamma_{s-r}$. If we use the exact value of $\gamma_c$, the simulation results will be further closer to analytical ones. The exact $\gamma_c$ can be calculated from the exact bit error probability of the convolutional code [23], but the expression is very complex.1

Figs. 9-11 compare the throughput of these DTC schemes. It can be noted that the throughput of DTC-SIR is higher than both the DTCs with and without ARQ at $\gamma_{s-r} = 10dB$. The throughput gap between them decreases as $\gamma_{s-r}$ increases. Therefore although the BER performance of DTC-SIR is worse than that of DTC with ARQ, the former will significantly improve the throughput of a distributed coding system.

B. Comparison for various SNR gaps between $\gamma_{s-r}$ and $\gamma_{r-d}$

It can be noted from (42) that for a certain code if we set the $SNR_{\text{Gap}}$ as a constant, $SNR_{\text{loss}}$ will become a constant for all $\gamma_{r-d}$, then the BER curve of the DTC-SIR should be parallel to that of perfect DTC and the gap between them is the $SNR_{\text{loss}}$ given in (53).

We assume that the signal energy decays exponentially with distance between two nodes as shown in Section 2. Obviously,
SNR\textsubscript{Gap} is determined by the ratio of power transmitted from the source and the relay, as well as the relative distance of \(d_{sr}\) and \(d_{rd}\). More specifically,

\[
\text{SNR}_{\text{Gap}} = 10 \log \left( \frac{P_s (G_{sr})^2}{P_2 (G_{rd})^2} \right) = 10 \log \left( \frac{P_s}{P_2} \left( \frac{d_{sr}}{d_{rd}} \right)^{-\kappa} \right)
\]

where \(P_s/P_2\) and \((d_{sr}/d_{rd})^{-\kappa}\) are referred to as the power amplification factor and the geometrical distribution factor, denoted by \(\rho_p\) and \(\rho_d\), respectively.

\(\rho_p\) denotes the ratio of the source and relay transmit power while \(\rho_d\) represents the ratio of \(d_{sr}\) and \(d_{rd}\), which depends on the network geometrical distribution. In this sub-section, we evaluate the performance of DTC-SIR for various SNR gaps to investigate the effect of power amplification factor and the network geometrical distribution on system performance and throughput.

Figs. 12-13 show the BER performance comparison for various SNR gaps. It can be observed that DTC-SIR significantly outperforms DTC with no ARQ. The performance of the DTC-SIR is worse than that of DTC with ARQ by about 1.5dB and 2dB, at SNR\textsubscript{Gap} of 0dB, at the BER of \(10^{-4}\).

From (53), we can also calculate that the SNR\textsubscript{loss} for SNR\textsubscript{Gap} of 0dB and 4dB are 1.46dB and 0.64dB, respectively. Fig. 14 compares the analytical SNR loss derived from (53) and the one obtained from simulation results in Figs. 12-13. From the figure, we can observe that the simulation results are consistent with the analytical results. The SNR\textsubscript{loss} is almost a constant for a fixed SNR\textsubscript{Gap} and the difference between the analytical results and simulation results is within 0.3dB.

Figs. 15-16 compare the corresponding throughput of these schemes. From the figures, we can note that the throughput of DTC-SIR is higher than both the DTCs with and without ARQ.

As shown in (54), the SNR\textsubscript{Gap} is determined by the power amplification factor \(\rho_p\) and the geometrical distribution factor \(\rho_d\). In order to increase the SNR\textsubscript{Gap} enabling DTC-SIR to approach the perfect DTC, we should increase \(\rho_p\) and \(\rho_d\). This can be achieved by placing the relay closer to the source than to the destination and/or making the transmit power from the source larger than that from the relay.

VII. CONCLUSION

In this paper, we present a new distributed turbo coding (DTC) scheme. In the proposed scheme, the relay transmits the soft information of the codeword. Analytical results demonstrate that imperfect decoding at the relay has only a slight effect on the system performance and that the proposed scheme approaches the performance of perfect DTC. Simulation results confirm that the proposed scheme also approaches the performance bounds of a distributed 2-hop relay network at high SNR.

REFERENCES

Fig. 15. Throughput comparison at $SNR_{Gap}=0dB$

Fig. 16. Throughput comparison at $SNR_{Gap}=4dB$


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