On the Performance of Space Time Turbo Trellis Codes with Adaptive Power Allocation

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Abstract—It has been shown that when partial or full channel state information (CSI) is available at the transmitter, the performance of a space time coded system can be dramatically improved by weighting the transmitted signals. In this letter, we evaluate the performance of space time turbo codes with adaptive power allocation when only partial CSI is available at the transmitter. An optimum power allocation strategy for space time turbo trellis coded systems (STTTC) is derived by simulations. Simulation results show that the proposed power allocation can bring a gain of about 0.8dB gain relative to the conventional STTTC with equal power allocation.

Index Terms—Space time turbo codes, partial channel feedback, adaptive power allocation.

I. INTRODUCTION

SPACE-TIME codes [1], [2] have been proposed for a system with no channel state information (CSI) available at the transmitter to achieve both coding and diversity gains. If partial or full CSI is made available at the transmitter, the system performance and capacity can be further enhanced [3]–[5]. Space time codes combined with dynamic power allocation were considered in [4] and [5], respectively. It was shown that compared to the conventional space time codes with equal power allocation, the adaptive power allocation can bring system a significant gain.

The techniques combining space time codes and turbo coding schemes have been of great interest recently [6]–[8]. Bandwidth efficient full rate space-time turbo trellis coding schemes (STTTC) were developed in [6], [7]. The coded sequences were constructed by using symbol interleaving and alternative parity symbol puncturing. A full rate space-time turbo code - assembled space time turbo trellis code (ASTTTC) was developed recently in [8]. It was shown that such scheme outperforms the other space time turbo coding schemes [6], [7] over fast fading channels.

Most of the published papers for space time turbo coded system assume that the transmitter has no CSI. In this letter, we consider a system for which partial CSI is available at the transmitter and design a power allocation strategy for a space time turbo coded system. The optimum power allocation strategy is obtained by simulations. Simulation results reveal that the overall gain of the STTTC with adaptive power allocation relative to the conventional STTTC with equal power allocation consists of a weighting and a coding gain. For a fixed code, there is a tradeoff between the weighting and coding gains, that is, increasing the weighting gain will decrease coding gain, while increasing the coding gain will decrease the weighting gain. The optimal power allocation for STTTC should be found by maximizing the sum of the weighting and coding gains. Simulation results exhibit that the proposed power allocation scheme can bring a gain of 0.8dB gain relative to the conventional equal power allocation over both quasi-static and fast fading channels.

II. SYSTEM MODEL

It was shown in [8] that an assembled space time turbo trellis code (ASTTTC) outperforms other existing space time turbo coding schemes over fast fading channels. As an example, this letter only considers the ASTTTC with adaptive power allocation, referred to as an ASTTTC-APA. The proposed method can be applied to any other space time turbo coding schemes in a straightforward manner. Since most of published papers on space time turbo codes consider the system with two transmit antennas, this letter also concentrates on the system with two transmit antennas.

The block diagram of an ASTTTC-APA system is shown in Fig. 1. The binary input sequence, denoted by $C$, is encoded by two parallel concatenated convolutional encoders, separated by an interleaver. The four encoder outputs, denoted by $V_i$, $i = 1, 2, 3, 4$ are modulated by respective QPSK modulators. The modulated symbol in streams $i$ at time $t$ is denoted by $s_i(t), i = 1, 2, 3, 4$.

Symbols $s_i(t), i = 1, 2, 3, 4$ are assembled into two symbols, denoted by $\tilde{S}_1(t), i = 1, 2$, by using a pre-defined function $f(x, y)$:

$$\tilde{S}_1(t) = f(s_1(t), s_3(t))$$

$$\tilde{S}_2(t) = f(s_2(t), s_4(t))$$

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We assume that the transmitter knows the ordinal number of channel quality for each transmit antenna, i.e., which antenna has the best channel quality, which one is the second best and so forth. The channel quality for transmit antenna \(i\) is described by \(\sum_{j=1}^{n_R} |h_{ij}(t)|^2\), where \(h_{ij}(t)\), \(i = 1, 2, j = 1, \ldots, n_R\), is the fading coefficient between the transmit antenna \(i\) and the receive antenna \(j\) at time \(t\) and \(n_R\) is the number of receive antennas. Without loss of generality, in this paper we assume that

\[
\sum_{j=1}^{n_R} |h_{1j}(t)|^2 \geq \sum_{j=1}^{n_R} |h_{2j}(t)|^2
\]

(3)

Based on the ordinal number of channel quality, the assembler outputs \(S_1(t)\) and \(S_2(t)\) are allocated different powers \(P_1\) and \(P_2\), respectively, and then radiated through two transmit antennas. As shown in Fig. 1, in this letter, we assume that \(S_1(t)\) is always transmitted through the best antenna\(^1\). Furthermore, \(P_1\) and \(P_2\) satisfy the overall power constraint,

\[
\sum_{i=1}^{2} P_i = P.
\]

The received signal at time \(t\) at receive antenna \(j\) can be written as

\[
r_j(t) = h_{1j}(t)\sqrt{P_1}S_1(t) + h_{2j}(t)\sqrt{P_2}S_2(t) + n_j(t)
\]

(4)

where \(n_j(t)\) is an independent zero mean complex Gaussian random variable with the two sided noise power spectral density of \(N_0/2\) per dimension.

### III. Space Time Turbo Trellis Codes with Adaptive Power Allocation

In this section, we evaluate the performance of the ASTTTC-APA and derive the optimum power allocation strategy by simulations. For the sake of simplicity, we consider a system with one receive antenna. All simulations were performed for a frame size of 130 symbols and QPSK modulation with spectral efficiency of 2 bit/s/Hz. An S-Random interleaver is chosen for the turbo coder and the depth of interleaver size is 512 symbols\(^2\). We applied a 4-state recursive systematic convolutional (RSC) code with generator matrix

\[
\begin{pmatrix}
1 & 1 + D + D^2 \\
1 & 1 + D + D^2
\end{pmatrix}
\]

as the component code. A quasi-static fading channel is assumed, for which the fading coefficients are constant within one interleaver block of 512 symbols but change independently from one block to another, as well as an independent fast fading channel, for which the fading coefficients change independently from one symbol to another. This channel model can be implemented by introducing a channel interleaver/de-interleaver pair at the transmitter and receiver.

For simplicity, we assume that \(f(x, y)\) is a linear function of \(x\) and \(y\), i.e.,

\[
f(x, y) = ax + by
\]

(5)

where \(a\) and \(b\) are referred to as the assemble coefficients, satisfying \(a^2 + b^2 = 1\), which ensures that the average power remains constant after the transformations in (1) and (2).

It has been shown in [8] that the ASTTTC achieves the optimum performance over quasi-static fading channels when

\[
(a, b) = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)
\]

(6)

In all simulations of this letter, the transformation function \(f(x, y)\) is of the form in (6).

We define the power allocation ratio \(\rho\) as

\[
\rho = \frac{P_1}{P_1 + P_2} = \frac{P_1}{P}
\]

(7)

where \(P_1 + P_2 = P\).

Fig. 2 illustrates the FER performance of the ASTTTC-APA for various power allocation ratios at different SNRs over quasi-static and fast fading channels. It can be noted from the Fig. 2 that the optimal \(\rho\) is not sensitive to the SNR. We observe that the optimum power allocation for ASTTTC over quasi-static fading channels is around \(\rho = 0.9\) while the optimum power allocation over fast fading channels is around \(\rho = 0.8\). Fig. 3 compares the FER performance of the ASTTTC with equal and optimum power allocations over quasi-static and fast fading channels. It shows that the ASTTTC with the optimum power allocation is superior by about 0.8dB to the conventional ASTTTC with equal power allocation at the FER of 1.0e-2.

We define the weighting gain of the ASTTTC-APA as the received SNR gain of the ASTTTTC-APA relative to...
various plots the overall gain, weighting and relative coding gains for gain from the total gain obtained from simulations. Fig. 4 the weighting gain for various power allocation ratios and we of a weighting and a coding gain. From (9), we can calculate conventional ASTTTC with equal power allocation consists ASTTTC-APA, denoted by $w_i$ where $h_{ij}$ is the fading coefficient between the transmit antenna $i$ and the single receive antenna, $p(|h_1|, |h_2|) = 4|h_1||h_2|\exp(-|h_1|^2 - |h_2|^2)$ is the joint distribution of $|h_1|$ and $|h_2|$ [9], $\gamma_{eq}$ is the received SNR for space time coded system with equal power allocation ($\rho = 0.5$).

From (8), we can calculate the weighting gain of the ASTTTC-APA, denoted by $w(\rho)$, as

$$w(\rho) = 10\log \left( \rho + \frac{1}{2} \right) \text{ (dB)}$$  \hspace{1cm} (9)

The overall gain of the ASTTTC-APA relative to the conventional ASTTTC with equal power allocation consists of a weighting and a coding gain. From (9), we can calculate the weighting gain for various power allocation ratios and we can get the relative coding gain by subtracting the weighting gain from the total gain obtained from simulations. Fig. 4 plots the overall gain, weighting and relative coding gains for various $\rho$ over fast fading channels.

It can be noted from Fig. 4 that the weighting gain is a monotonically increasing function of $\rho$, while the coding gain is a monotonically decreasing function of $\rho$. The ASTTTC-APA with $\rho = 1$ can achieve a maximum weighting gain, but has the minimum coding gain, while the conventional ASTTTC with equal power allocation ($\rho = 0.5$) has a maximum coding gain, but achieves the minimum weighting gain. This implies that adaptive power allocation reduces the coding gain of a space time coded system, while bringing system a weighting gain. The optimum ASTTTC-APA can obtain neither the maximum weighting gain, nor the maximum coding gain, but the maximum overall gain.

IV. CONCLUSION

This letter evaluates the performance of a space time turbo trellis coded system with adaptive power allocation. The optimum power allocation is derived from simulations. Simulation results show that the space time turbo trellis codes with optimum power allocation considerably outperform the conventional space time turbo trellis codes with equal power allocation. As an example, the letter only considers the assembled space time turbo trellis codes. The same method can be applied to any other space time turbo coded system in a straightforward manner.

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